PRODUCTION OF REGIONAL SINGLE-ENTRY VOLUME TABLES AND DEVELOPMENT OF RAW WOOD PRODUCT MIX MODEL FOR ONTARIO

by

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A Graduate Thesis Submitted
In Partial Fulfillment of the Requirements
for the Degree of Master of Science in Forestry

Faculty of Forestry
Lakehead University
October, 1995

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ISBN 0-612-09220-8

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AND

DEVELOPMENT OF RAW WOOD PRODUCT MIX MODEL FOR ONTARIO

FACULTY OF FORESTRY
LAKEHEAD UNIVERSITY
THUNDER BAY, ONTARIO

by

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A CAUTION TO THE READER

This M.Sc.F. thesis has been through a semi-formal process of review and comment by at least two faculty members.

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ABSTRACT

Li, Guohua. 1995. Production of regional single-entry volume tables and development of raw wood product mix model for Ontario. 73 pp + appendices. Advisor: Dr. Hugh G. Murchison.

Key Words: single-entry volume equations, height-diameter functions, stem profile equations, wood product mixes.

Regional single-entry volume equations for northern Ontario were derived based on the standard volume equations by Honer *et al.*. A site stratification methodology was employed to derive localized regional height-diameter equations. Using this method, the variation of height prediction for a given species within a region was greatly reduced. Thus, site specific equations were derived for each species.

For stand data lacking tree height information, the exponential function: $\text{Height} = b_1 \times \exp(b_2/\text{Dbh})$ proved best for height prediction. This model was used to substitute height in standard volume equations. In addition to the total volume and gross merchantable volume based on top diameter and stump height, the net merchantable volume based on age was also derived.

Stem profile equations were also fitted and used to model wood product mixes. The results showed that Max and Burkhart's model was the most accurate and precise model in predicting top diameters and section heights along the bole, while the model by Demaerschalk performed better for volume prediction. These stem profile equations demonstrated maximum flexibility in dealing with the wood product mixes. By combining the stem profile models and the single-entry volume equations, a modelling system was developed to estimate wood product mixes for stands based on dbh distributions. The wood product mix model developed can be used at both the tree level and stand level. A Fortran program was written to facilitate the calculations for modelling the combinations of wood product mixes at the stand level.

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ACKNOWLEDGEMENTS

This thesis would not be possible without the data provided from various individuals and agencies across Ontario. My thanks to Northeastern Ontario Science and Technology for the assistance in providing the Northeastern region data set, to Northwestern Ontario Science and Technology for the Northwestern region data set, to Ontario Forest Research Institute for the FMI data bank, and to many others for other data sets. I am grateful for their support.

This project was funded and supported by Ontario Ministry of Natural Resources through Ontario Forest Institute, Ontario Lumber Manufacturers Association and Northern Ontario Heritage Fund Corporation, and Lakehead University.

I wish to thank all the members of my advisory committee for their many efforts in encouraging, supporting and correcting the errors in the first few drafts. Dr. R. E. Pulkki made many good comments and suggestions for this thesis. In particular, I wish to thank Dr. Hugh G. Murchison who gave me the tremendous supervision, guidance and encouragement which made this research work easier.

I also wish to thank Dr. Harry V. Wiant. His comments and suggestions were greatly appreciated.

INTRODUCTION

PROBLEMS

An estimate of tree volume is required in almost all aspects of forest activities. Local volume information is an immediate need for forest management throughout Ontario. This need is identified in "Forest Growth and Yield: A Master Plan for Ontario" (OMNR, 1992).

Standard volume equations with merchantable volume conversion factors have been developed and cover most commercial tree species in Ontario (Honer *et al*., 1983; Honer, 1967, 1964). These volume equations employ two input variables: diameter at breast height (dbh) and total height. In some forest inventory practices, especially in low intensity timber cruises, the requirement for height information, which is more difficult and costly to collect than dbh, may constitute a problem. Simple, yet reliable, single-entry volume tables (or local volume tables) with dbh as the only entry variable may provide a solution.

Tree height is usually a function of dbh. This function can be used to access multiple-entry volume tables for the construction of single-entry volume tables which give tree volume as a function of dbh (Murchison, 1984). It can also be used to predict height when the wood product volumes are estimated for a stand using only dbh distributions. Maurer (1993) demonstrated that for local applications, derived height equations can be substituted for the height variables in the standard volume equations (Honer *et al.*, 1983) to produce tree volume equations.

Traditionally, according to Maurer (1993), single-entry volume tables relate diameter to merchantable volume derived from roadside scaling measurements. Many equations would be needed to just cover Northeastern Ontario, as each equation expresses a species' site specific volume relationship, and is related to a license area or township. It is costly, inefficient and very impractical to develop the many single-entry volume tables required for each specific area.

The existing data from many previous research projects and operational surveys throughout the province is an excellent data source. These data sets can be collated to produce regional and provincial single-entry volume tables.

Empirical data can provide some information about wood products which can be produced from individual trees or specific stands. However, such information will generally be restricted to existing product specifications (Martin, 1981). To acquire a thorough knowledge of the multiple-product information for individual trees and stands, a wood product mix modelling system is needed that provides flexibility with changing dimensional requirements of products and gives reliable volume estimates.

The author had little success in locating literature dealing with wood product modelling. One of the major purposes of this study is to develop a system to model wood products at both tree and stand level through the use of stem profile models. Using modelling, it is possible to estimate the portions and combinations of various wood products within a single tree or a stand according to the desired merchantable specifications. This estimation is directly related to dbh and site class when available.

OBJECTIVES

There were three objectives for this study. The primary objective was to use stem analysis, growth and yield, and operational cruise (OPC) data gathered within Ontario to derive regional and provincial single-entry volume tables for all species for which adequate data is available. The second objective was to develop a system to model raw wood product mixes (i.e., combinations of veneer bolts, sawlogs, pulpwood and chips), based on stem analysis, scale returns, OPC and local volume table (LVT) data. This model will allow wood product estimation based on annual work schedule (AWS) and forest resource inventory (FRI) information. The third objective was to transfer this technology to Ontario Ministry of Natural Resources (OMNR) staff. In order to accomplish the above objectives, the following tasks were required:

1. gather as many data sets as possible from within the regions of Ontario,
2. test and select the best possible models (including height prediction models, age prediction models and stem profile models),
3. derive site-stratified regional and provincial single-entry volume equations from standard volume equations and tabulate these equations in forms suitable for field applications,
4. make comparative analyses of the regional and provincial single-entry volume equations,
5. develop a system based on stem profile models to model wood product mixes, and
6. model dbh distributions found in merchantable stands,

7. model recovery rates by wood product mixes based on OPC and FRI descriptions.

LITERATURE REVIEW

VOLUME TABLE CONSTRUCTION

Individual tree volume cannot be directly measured in the field, but must be estimated through the use of ancillary variables (Murchison, 1984). Husch *et al.* (1982) classified volume determination methods as: standard formulae, integration, liquid displacement and graphical estimation. Total tree volumes are usually estimated using volume equations (Munro and Demaerschalk, 1974; Cao *et al.*, 1980). These equations customarily predict tree volumes from dbh, and either total or merchantable height.

Tree volume tables have been constructed using many different approaches. The preferred method for constructing multiple-entry volume tables is by regression analysis (Avery and Burkhart, 1983). The volume-ratio approach has been used to develop volume tables by some mensurationists (Honer, 1964, 1967; Honer *et al.*, 1983; Burkhart, 1977). This approach is flexible in estimating both total and merchantable volume with varying utilization standards. Another way of addressing tree volume is by stem profile equations or taper equations (Demaerschalk, 1972; Cao *et al.*, 1980; Avery and Burkhart, 1983; Alemdag, 1988; Czaplewski *et al.*, 1989a, 1989b; Czaplewski and Bruce, 1990). Stem profile models provide the maximum flexibility for computing volumes of any specified portion of a tree bole. Stem analysis can also generate accurate volume estimate (Kavanagh, 1983; Biging, 1988; Maurer, 1993).

Importance sampling was introduced by Gregoire *et al* (1986), which provided unbiased estimates of tree volume. This estimate is based on one diameter measurement, the height of the point of measurement being selected randomly proportional to the estimated distribution of volume along the bole as determined by a proxy taper function. This volume estimate is then adjusted by the ratio of the cross-sectional area measured at the sample point to that predicted by the proxy function. Wiant *et al* (1989) applied importance sampling to a radiata pine stand and found that importance sampling reduced dendrometry by 96% compared to using 3P sampling. Wood and Wiant (1990) demonstrated that centroid sampling, a variant of importance sampling, is superior to Huber's formula for estimating log volume based on a single measurement of diameter.

Standard and single-entry volume tables are the most commonly used volume tables (Maurer, 1993). Standard volume tables usually employ two variables (dbh and total tree height, and sometimes with stem form as an additional variable). The principal variables ordinarily associated with tree volume are dbh and tree height (Chapman and Meyer, 1949; Husch *et al.*, 1982; Avery and Burkhart, 1983). Tree form is also an important variable in predicting tree volume (Avery and Burkhart, 1983). Flewelling (1993) pointed out that stem form differences cause volume computations based on dbh and total height to be in error. Single-entry volume tables are constructed based on the single variable of dbh (Avery and Burkhart, 1983). Chapman and Meyer (1949) stated that even trees of the same species, with identical dbh and total heights, do not necessarily have the same volume. A single universal volume table that would apply to all conditions and species is therefore not possible.

Foresters are often more interested in estimating merchantable volume, that is, the content of tree boles from a given stump height to some fixed top diameter or height limit (Cao *et al.*, 1980; Alemdag, 1990). Honer (1967) used a volume ratio approach to estimate total tree volume along with a merchantable volume conversion factor. Burkhart (1977) introduced a merchantable volume equation to provide estimates of the ratios of merchantable to total volume.

Honer (1964) emphasized the importance of flexibility of any system developed to estimate merchantable volume. Alemdag (1990) summarized that taper curves, volume equations for a given diameter of utilization, and ratio expressions for variable merchantable diameters and merchantable heights are the three main approaches to estimate merchantable volume.

One way of constructing single-entry volume tables is from the scaled measure of felled trees (Avery and Burkhart, 1983). Single-entry volume tables can also be constructed from existing multiple-entry volume tables (Chapman and Meyer, 1949; Avery and Burkhart, 1983; Maurer, 1993). Husch *et al.* (1982) also pointed out that single-entry volume tables are normally derived from standard volume tables. In constructing single-entry volume tables from standard volume tables, tree height information must be estimated in relation to tree diameters (Husch *et al.*, 1982).

In constructing single-entry volume tables in Northeastern Ontario, Maurer (1993) made an effort to reduce the variation of height-diameter relationships by stratifying the height functions by site class. He concluded that using local height equations to drive existing standard volume equations provides an efficient way to produce local volume information.

Chapman and Meyer (1949) argued that although the total or merchantable height may vary considerably, even in the same area, the height curve based on diameter can be used to represent the local condition in the construction and application of the single-entry volume tables. Avery and Burkhart (1983) cautioned that the labels "local" and "standard" are often misleading, for they tend to imply that single-entry volume tables are somehow inferior to standard volume tables. Such an assumption is not necessarily true, particularly when the single-entry table in question is derived from a standard volume table.

Gillis and Edwards (1988) pointed out that theoretically, tree volume equations constructed from regression analysis should only be applied to that portion of the forest from which they were derived. In practice, however, equations are applied regionally with the assumption that the local fit is acceptable. It is not essential that single-entry volume equations be applied to relatively small areas (Avery and Burkhart 1983).

Usually regional tree height functions can be used to access standard volume tables. Murchison (1984) pointed out that one use of tree height functions is in the construction of single-entry volume tables which give tree volumes as a function of diameters. Bonner (1974) concluded that the errors due to tree height estimation instead of direct height measurement are minor and do not significantly affect volume estimates at stand level.

Numerous height-diameter regression models have been proposed and used in the past. Avery and Burkhart (1983) suggested that the exponential model of the form Height = $b_1 \times \exp(b_2/\mathrm{Dbh})$ is satisfactory for a wide range of species and for both total and merchantable tree height. Maurer (1993) compared the fit of three models to data sets collected in Northeastern Ontario and concluded that the performance of the above model is superior to those of the basal area model and the linear model. Arabatzis and Burkhart (1992) compared eight height-diameter models and demonstrated that the above exponential model performed the best, especially when fitted to the data selected by simple random sampling. Huang *et al.* (1992), on the other hand, compared the performances of 20 nonlinear height-diameter models for major species in Alberta and found the Chapman-Richards height-diameter model to be one of the most accurate height prediction models for major Alberta tree species.

STEM PROFILE MODELLING

A stem profile model is basically a description of the stem profile in terms of diameters and heights along the bole (Alemdag, 1988). The major applications of stem profile models include:

1. to predict diameter at any height along the main stem (Cao, *et al.*, 1980; Martin, 1981, 1984; Czaplewski *et al.*, 1989a, 1989b),
2. to predict height to any specific diameter limit (Martin, 1981, 1984),
3. to estimate total volume and merchantable volume by integration (Demaerschalk, 1972; Cao *et al.*, 1980; Avery and Burkhart, 1983; Alemdag, 1988; Czaplewski *et al.*, 1989a, 1989b; Czaplewski and Bruce, 1990; Flewelling, 1993),
4. to estimate the segment volume between any two heights on the bole (Martin, 1981), and
5. to test the accuracy of volume equations (Biging, 1988).

There have been numerous approaches to model stem form in recent decades. According to Sterba (1980), the first stem profile model was introduced by Behre in 1923. Since then, many stem profile models have been developed. Kozak (1988) summarized that stem profile equations reported in the literature can be divided into two major groups: i.e., the single taper equations and the segmented taper equations. For the single taper equations, the major weakness is the significant bias in estimating diameters close to the ground as well as at some other parts of the tree. Their advantages are that they are easy to fit, usually easy to integrate for volume calculation and easy to rearrange for calculation of merchantable height. With the segmented taper equation, the bias for diameter predictions are greatly reduced, especially at the butt portion of a tree bole (Martin, 1981). This results in more accurate estimations of volume and height. The disadvantages are that in most cases, the parameters are difficult to estimate and the formulae for calculating volume and merchantable height are cumbersome or nonexistent. Byrne and Reed (1986) observed that complex stem profile equations, such as the segmented taper equations, provide better fit of the stem profile than single taper equations, especially in the high volume butt region.

Kozak *et al*. (1969) proposed a simple quadratic model for describing stem taper of many tree species in British Columbia. Ormerod (1973) even proposed a very simple equation in which only one coefficient was involved. Demaerschalk

(1972) derived a compatible stem profile equation from a logarithmic volume equation in that both the stem profile equation and the volume equation yield the same results. Max and Burkhart (1976), on the other hand, introduced a complicated segmented polynomial model including two join points, two dummy variables and four regression coefficients.

Based on the assumption that a tree stem can be divided into three geometric shapes, Max and Burkhart (1976) developed three separate submodels that describe the neiloid frustum of the lower bole, the paraboloid frustum of the middle bole, and the conical shape of the upper portion. The three submodels are then spliced together at two "join points" into an overall segmented polynomial tree model (Martin, 1981).

Grosenbaugh (1966) gave a comprehensive and detailed account of tree form. It is generally agreed that there are wild variations in the stem form due to variations in the rate diameter decreases from the butt to the tip (Husch *et al.*, 1982). Grosenbaugh (1966) pointed out that the stem shapes assume an infinite variation along the stem and numerous paired measurements of height and diameter would be required to describe the entire stem. Demaerschalk and Kozak (1977) suggested that the use of different models for the lower and upper bole could improve the prediction system considerably.

In the past, stem profile models were fitted with both diameter (d) and squared diameter (d^2). When d is fitted as the dependent variable, the model does not provide optimum estimates of volume (Demaerschalk, 1972). The stem profile model with d^2 as the dependent variable, on the other hand, tends to over-estimate stem diameters as the result of retransformation, thus leading to the over-estimation of volume (Czaplewski *et al.*, 1989a).

Demaerschalk (1973) pointed out that an equation which is best for taper is not necessarily best for volume. In spite of this drawback, stem profile models are still widely used for estimating volumes, and are especially useful when dealing with wood product mixes (Martin, 1981).

WOOD PRODUCT MIXES MODELLING

The specifications for wood products and utilization standards can evolve rapidly, often in response to local market and economic conditions (Czaplewski *et al.*, 1989b). The traditional volume equations are no longer sufficient to meet the needs of estimating the volumes for the varying wood products (Martin, 1981). Stem profile models can be used to provide greater flexibility with changing specifications and with new products (Martin, 1981). McTague and Bailey (1987) insisted that for the purpose of merchandising the tree into multiple products, the development of a stem profile function is essential.

ACCURACY AND PRECISION

Two major sources significantly affecting the accuracy of models include the choice of equation used in determining volume, and the error in measuring diameters and lengths of logs (Biging, 1988). Reynolds (1984) pointed out that one method of determining how well a model will perform is to compare predictions from the model with an existing model or with actual values from the real system.

Reynolds (1984) expanded on Freese's (1960) accuracy test by presenting a complete system for testing accuracy. Rauscher (1986) developed a BASIC program (ATEST) to facilitate implementation of Reynolds' system. Based on Rauscher's program, Wiant (1993) developed a DOS-based program (DOSATEST). DOSATEST is very handy for comparing results.

Three regression statistics were widely employed to describe the fit of models (Czaplewski *et al.*, 1989a; Martin, 1981; Kozak *et al*, 1969). These statistics include:

1. mean squared error (MSE),
2. coefficient of determination (r^2),
3. standard error of estimate (SE).

In evaluating accuracy and precision, some mensurationists (Cao *et al.*, 1980; Martin, 1981, 1984) employed the following criteria:

1. bias (the mean of differences between the actual and predicted values),
2. mean absolute difference (the mean of the absolute differences),
3. standard deviation of the differences (SD).

Methods of ranking were also used to evaluate the performance of models in their prediction ability (Cao *et al.*, 1980; Martin, 1981, 1984). A rank number is assigned so rank number one corresponds to the model which has the smallest absolute value of the criteria (i.e., bias, mean absolute difference, and/or standard deviation) being used (Cao *et al.*, 1980). Therefore, the smaller the rank number, the better the model.

Huang *et al.* (1992) employed three criteria (asymptotic t-statistics, MSE and the plot of studentized residuals against the predicted value) to judge the performance of height-diameter functions. They further pointed out that for any appropriate function, the asymptotic t-statistics for each coefficient should be significant, the model MSE should be small and the studentized residual plot should show approximately homogeneous variance over the full range of the predicted values.

METHODOLOGY

DATABASE AND DATA PROCESSING

The database used in this study includes two basic data types. The first type was growth and yield data from Northeastern and Northwestern regions of Ontario. These data sets represent a wide range of species compositions, stand structure and densities, diameter, height, age groups, and site conditions. They include basic attributes such as species, dbh, total height and total age. They were used to develop the single-entry volume tables for these two regions. The summary information for these data sets are shown in Tables 1 and 2.

The second type includes stem analysis data from the Northwestern (Table 3) and Central Ontario Regions (Table 4), and from the Forest Management Institute (FMI) data bank (Table 5).

Table 1. Summary information of tree data for Northeastern Ontario

Species*	Trees	Dbh (cm)			
		Mean	Min.	Max.	Std. Dev.
Black ash	19	24.6	13	38	6.7
Trembling aspen	1404	29.4	2	64	10.0
Balsam fir	612	16.1	3	47	6.1
White birch	1023	21.4	4	57	7.5
Yellow birch	35	32.7	19	66	11.0
White cedar	392	27.2	4	73	11.1
Tamarack	108	21.4	3	44	9.0
Red maple	76	18.1	4	42	7.5
Sugar maple	18	35.3	14	58	12.4
Balsam poplar	80	25.3	6	47	8.9
Jack pine	2038	24.7	2	51	7.1
Red pine	20	33.1	10	52	8.8
White pine	144	50.4	12	104	25.1
Black spruce	2170	18.2	2	45	6.1
White spruce	484	29.9	6	63	10.2

* see Appendix A for the full Latin name

Table 2. Summary information of tree data for Northwestern Ontario

Species	Trees	Dbh (cm)			
		Mean	Min.	Max.	Std. Dev.
Black ash	64	25.7	12.9	50.5	8.6
Balsam fir	1062	15.0	2.0	39.5	5.9
White birch	715	14.7	1.7	51.8	7.1
White cedar	109	24.2	6.2	51.3	8.4
Tamarack	72	22.1	6.6	50.8	8.5
Red maple	15	16.5	9.8	26.0	5.6
Balsam poplar	97	29.1	10.1	50.4	9.0
Jack pine	1903	22.2	4.0	83.0	6.8
Aspen (gen.)	1386	19.7	3.9	55.6	9.2
Red pine	255	34.8	12.2	61.3	8.4
White pine	157	35.9	11.8	74.0	11.9
Black spruce	2687	17.6	2.9	47.2	5.6
White spruce	589	23.2	4.0	55.3	10.2

Table 3. Summary information of stem analysis data for Northwestern Ontario

Species	Trees	Sections	Dbh (cm)			
			Mean	Min.	Max.	Std. Dev.
Aspen (gen.)	35	3341	20.3	0.4	58.3	11.8
Jack pine	325	6962	12.5	0.2	37.3	8.0
Black spruce	80	8111	11.8	0.2	27.3	6.1

Table 4. Summary information of stem analysis data for Central Ontario

Species	Trees	Sections	Dbh (cm)			
			Mean	Min.	Max.	Std. Dev.
White birch	16	340	18.3	14.8	24.4	3.1
Jack pine	311	5785	18.1	6.5	29.3	3.3
Red pine	232	4588	23.6	3.4	50.6	6.1
White pine	142	2872	25.2	16.8	46.9	4.9
Aspen (gen.)	213	4584	19.2	11.6	30.5	3.1
White spruce	54	982	23.0	15.2	35.2	4.7

Table 5. Summary information of FMI stem analysis data for Ontario

Species	Trees	Sections	Dbh (cm)			
			Mean	Min.	Max.	Std. Dev.
Largetooth aspen	65	603	23.1	5.1	36.7	7.0
Aspen (gen.)	396	6389	30.9	11.2	47.5	4.9
Trembling aspen	105	867	17.4	5.6	33.3	6.6
Beech	13	114	13.5	5.7	22.9	4.4
Balsam fir	43	496	11.2	4.2	23.1	4.9
White birch	75	592	14.1	5.6	24.5	3.9
White cedar	67	802	23.4	18.4	29.8	2.9
Jack pine	543	6470	20.7	4.5	164.8	8.3
Red pine	827	9734	29.9	5.3	65.5	13.3
White pine	904	10509	29.2	3.9	83.4	15.7
Black spruce	171	2051	14.2	5.1	30.5	5.3
White spruce	36	424	19.3	5.3	48.1	8.9

The data in Table 3 was measured and recorded by year with the latest year being 1991. The stem analysis data of Central Ontario consists of six species. Each tree is described by three different types of files which describe, for each individual tree, the general information, the disc information and the diameter information respectively. By combining these files, a new data file was created, which includes all the required information for the modelling exercises. The section measurements were made at one-metre intervals with the first measurement taken at 0.3 m from the butt. Smalian's formula (Husch *et al.*, 1982) was used to calculate a column of volumes in m^3. These volumes were added to the data file.

Most of the FMI data originated from a series of forest surveys carried out between 1918 and 1930, with some later additions. For most of the trees, information is available from the tree as a whole (such as dbh) and also from individual sections of trees. A detailed description of the FMI data can be found in "The Forest Management Institute Tree Data Bank" (MacLeod, 1978).

SINGLE-ENTRY VOLUME EQUATION DEVELOPMENT

With the localized height information, simple single-entry volume equations were derived from the existing standard volume equations (Honer *et al.*, 1983) for each species within a certain region which is claimed to have geographic similarity: i.e., the same height-dbh pattern throughout the region. In order to enhance this similarity, each species was further stratified by site class unless the sample size for a given species was small. In the later case, a combined "all sites" equation was constructed for a given species. As a result, a more site specific height-dbh relationship within the region was achieved. The procedures involved in developing the regional and provincial single-entry volume tables are summarized in Figure 1.

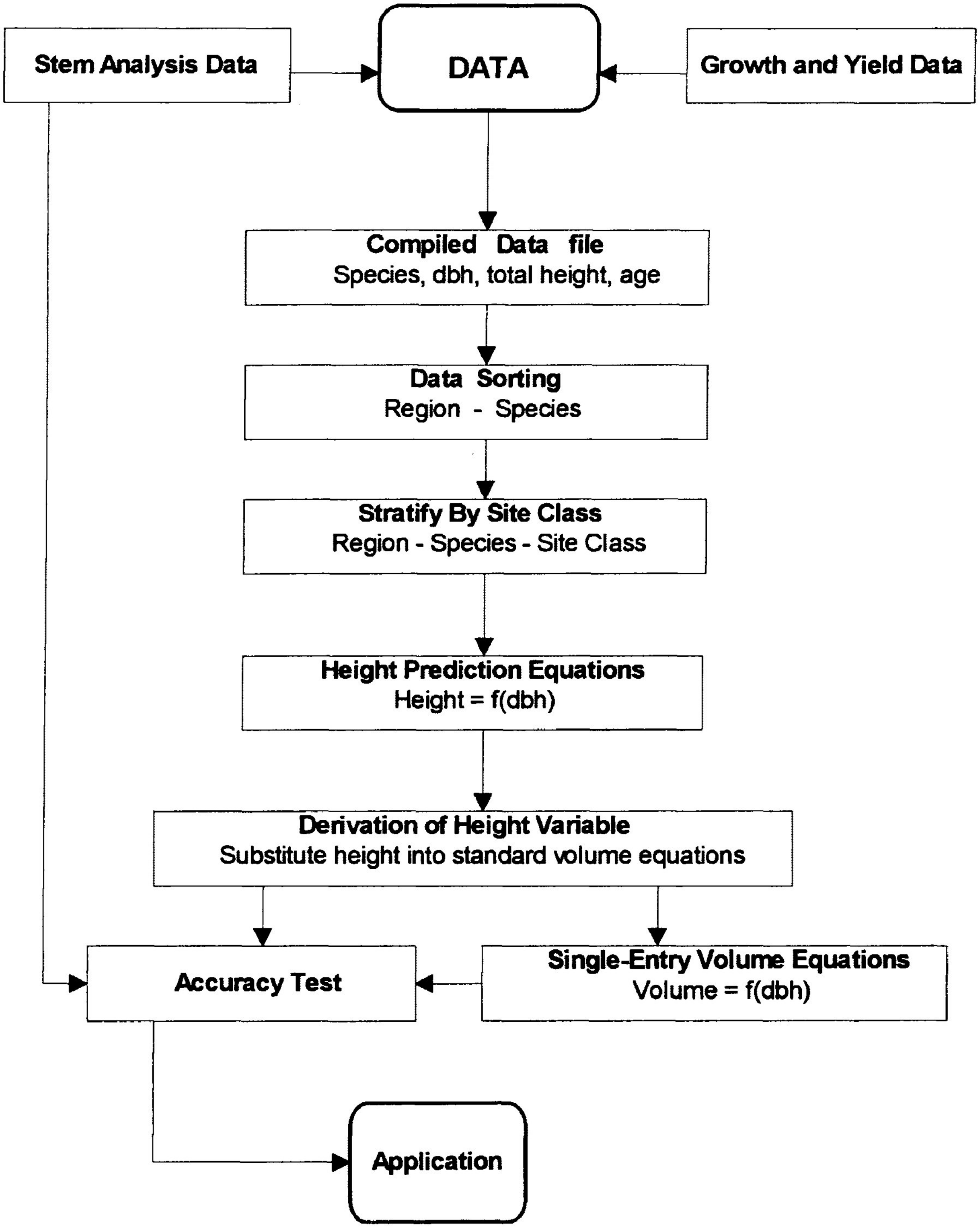


Figure 1. Flowchart of single-entry volume tables construction.

Data Screening

There usually exist some outlier cases in data sets. Some may be due to non-statistical reasons such as data recording and transcribing error. In order to exclude any bias for or against the analyzed equations, confirmed outliers were eliminated by employing the studentized deleted residual statistic (Weisberg, 1980; Myers, 1990) before the actual analysis was started.

Site Class Assignment

Trees in the same area may vary considerably in total or merchantable height within each diameter class (Chapman and Meyer, 1949). Such difference is commonly associated with changes in site quality (Chapman and Burkhart, 1949). Variation in height prediction can be reduced by stratifying each species by site class.

Each tree was assigned a site class using Normal Yield Tables (Plonski, 1981) based on total height and age. Site class assignment equations for the major tree species in Ontario were derived based on the mid-class height of each site class at the observed age (Maurer, 1993). Several sets of equations for different species or species groups are available for major tree species in northern Ontario (Appendix B).

For some species, the site-specific equations are impossible or inappropriate to derive. In these cases, data of the given species must be pooled together to derive an "all sites" equation. The following two cases are typical of the above:

1. the sample size for a given species is not large enough,
2. no appropriate site class equation is available for a given species.

Height Prediction Equations

Based on the relationship shown in Figure 2 (a), several mathematical models were tested and the most appropriate one was used in the subsequent analysis. Besides the two nonlinear height prediction models (Equations [1] and [2]) tested by Maurer (1993), two additional height-diameter models of the Chapman-Richards function (Equation [3], Huang *et al.*, 1992) and the quadratic form equation (Equation [4]) preferred by McDonald (1982) were further compared to the models. All the models were fitted using unweighted nonlinear least squares regression. The height prediction functions tested are listed below:

The exponential function

$$H = 1.3 + b_1 \times \exp (b_2/D) \qquad [1]$$

The basal area function

$$H = 1.3 + b_1 \times [1 - \exp (b_2 \times D^2)] \qquad [2]$$

Chapman-Richards function

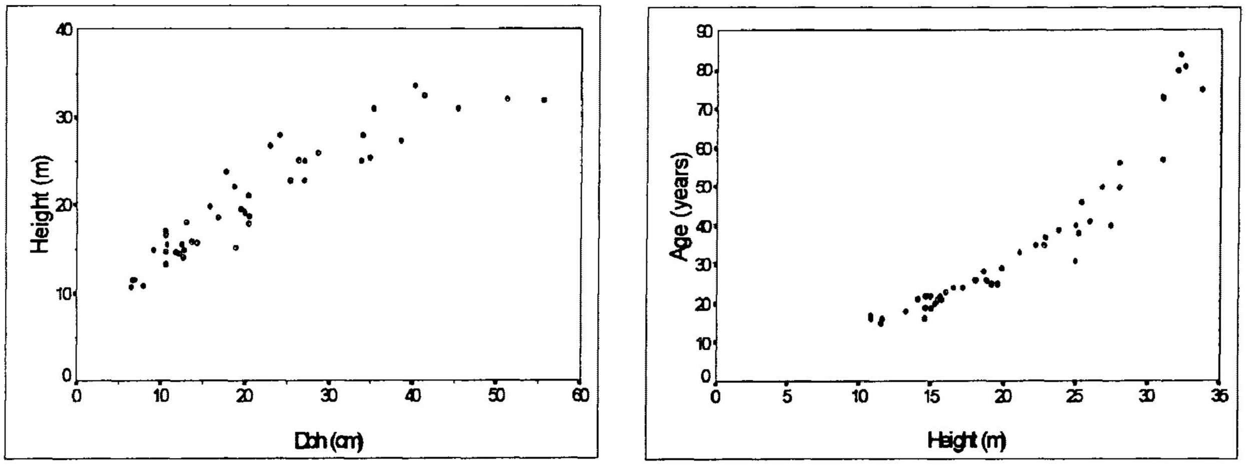
$$H = 1.3 + b_1 (1 - \exp(b_2 \times D))^{b_3} \qquad [3]$$

The quadratic function

$$H = b_1 + b_2 \times D + b_3 \times D^2 \qquad [4]$$

In the above equations:

H = total tree height from ground to tip,
D = diameter outside bark at breast height (1.3 m),
1.3 = constant used to account that dbh is measured at 1.3 m above ground.
exp = the natural logarithmic function (the base e=2.71828),
b_i = regression coefficients.



(a) (b)

Figure 2. Scatterplots showing (a) the height-dbh relationship and (b) the age-height relationship.

Age Prediction Equations

Age information is required in order to derive net merchantable volume. Tree height is usually closely related to tree age. Maurer (1994) proposed the following exponential-type function (Equation [5]) to predict tree age (A) from total tree height (H).

$$A = 1.3 + b_1 \times \exp(b_2 \times H) \qquad [5]$$

It appears that there is no reason to include the constant term 1.3 in the equation since the relationship between tree age and total tree height is assumed theoretically to pass through the origin (Figure 2 (b)). In this study, a modified Weibull function (Yang *et al.*, 1978) was adopted (Equation [6]).

$$A = b_1 \times \exp\ (b_2 \times H^{b_3}) \qquad [6]$$

Volume Equations

Total and Merchantable Volumes

The standard volume model by Honer *et al.* (1983) (Equation [7]) has been used extensively in Ontario. In this study, this model served as the base volume equation from which the single-entry volume equations (Equation [8]) were derived. The height variable in the standard volume equation (Equation [7]) was replaced by the locally derived height equation.

$$V = \frac{0.0043891 \times D^2 \times (1-0.04365 \times b_3)^2}{b_4 + \left(\dfrac{0.3048 \times b_5}{H}\right)} \qquad [7]$$

$$V_{SE} = \frac{0.0043891 \times D^2 \times (1-0.04365 \times b_3)^2}{b_4 + \left(\dfrac{0.3048 \times b_5}{1.3 + b_1 \times \exp\ (\frac{b_2}{D})}\right)} \qquad [8]$$

Where,

V_{SE} = single-entry volume (m^3)

b_1, b_2 = coefficients in Equation [1],

b_3, b_4, b_5 = coefficients in the standard volume equation.

The gross merchantable volume of a tree (V_{GM}), according to Honer *et al.* (1983), is calculated as a function of V_{SE} by excluding the top and stump portions of the tree.

$$V_{GM} = V_{SE} \times (r_1 + r_2X + r_3X^2) \qquad [9]$$

Where,

$X = (T^2/(D^2 \times ((1 - .04365 \times b_3)^2))) \times (1 + S/H)$

r_1, r_2, r_3 = regression coefficients

b_3 = regression coefficient in Alemdag and Honer's (1977) taper equation

T = top diameter

S = stump height

Net Merchantable Volume

The net merchantable volume is defined as the gross merchantable volume minus the cull volume. The percentage of cull volume in a tree is closely related to its age and site class. The net merchantable volume of a tree is therefore calculated using age-based cull equations applied to the gross merchantable volume. To derive the cull percentage equations for all species, three major sources of cull studies (OMNR, 1978; Basham, 1991; Morawski *et al.*, 1958) in Ontario were employed. The relationship between the percentage of cull wood volume and tree age can be expressed as:

$$V_{cp} = b_1 + b_2 \times A^{b_3} \qquad [10]$$

Where,

V_{cp} = percentage of cull wood volume in relation to merchantable volume,

b_1, b_2, b_3 = regression coefficients.

The net merchantable volume (V_{NM}) can be estimated as,

$$V_{NM} = \left[1 - \frac{V_{CP}}{100}\right] \times V_{GM} \qquad [11]$$

Comparisons of Regional and Provincial LVT's

The comparisons were conducted by regions to determine the necessity of separate equations for regions. The null hypothesis was, for a given species, that the coefficients (b_i) in Equation [1] are not significantly different for different regions and can be combined into one general equation. The test criteria (t) is expressed as (Myers, 1990):

$$t = (b_{ij} - b_{ik})/S.E.$$

Where:

b_{ij} = coefficient b_i for region j,

b_{ik} = coefficient b_i for region k,

S.E. = pooled standard error of coefficients b_{ij} and b_{ik}.

The hypothesis is:

$$H_0: \quad b_{ij} = b_{ik}, \quad \text{or } |b_{ij} - b_{ik}| = 0$$

$$H_1: \quad b_{ij} \neq b_{ik}, \quad \text{or } |b_{ij} - b_{ik}| \neq 0$$

STEM PROFILE MODELLING

Stem profile equations were fitted using the stem analysis data available for this study and used in the subsequent wood product mix modelling process. Several published stem profile models (Demaerschalk, 1972; Max and Burkhart, 1976; Kozak *et al.*, 1969; Ormerod, 1973) were examined and compared. These models are listed in Appendix C.

ACCURACY TESTING

For single-entry volume equations, accuracy testing was conducted by comparing the volumes estimated from the single-entry volume equations to the volumes from the standard volume equations (Honer *et al.*, 1983), as well as to the stem analysis volumes. The stem analysis volumes used for the testing were assumed to be the true values. The data sets for which the single-entry volume accuracy was tested come from the stem analysis data from Northwestern Ontario. Two species were made available for the testing: i.e., jack pine and black spruce.

All the tests were conducted by species and site class. A similar procedure was applied to test and compare the height predication equations and age prediction equations.

The stem profile models were evaluated in order to select the most appropriate one in the subsequent wood mix modelling process. Diameter prediction, height prediction and volume prediction by stem profile models were tested using sample data and independent data sets. For the three parameters (diameter, height and volume) tested, three criteria (Cao *et al.*, 1980) were employed.

In order to evaluate the overall performance of the stem profile models tested, a method of ranking was employed. For each individual tree species and each data set, a rank number was assigned to each model according to the three criteria. These ranks were then summarized for the three criteria for each species and for each data set. The overall ranking for each model was assigned based on the summed value for criteria, and for both the sample data and the independent data. The final overall ranking demonstrates the performance of a model compared with the others.

THE DIMENSIONAL REQUIREMENTS OF WOOD PRODUCTS

The dimensional requirements for wood products vary due to the impacts and constraints on forest industries. These include changes in technology in wood processing and handling systems, external economic influences on market demand, and changes in Provincial or industrial policies which impact on wood supply. Availability of raw material and accessibility to the mill are also important in acceptance of the raw material by each individual mill. Different mills utilize different species or groups of species depending on the products they produce. They set their own dimensional limits based on the type of equipment in use and the ability of this equipment to economically produce marketable products. Table 6 lists current dimensional requirements of roundwood from Northern Wood Preservers Inc., Thunder Bay, Ontario, and were used as the sample criteria for the wood product mix modelling in this study.

Table 6. A sample dimensional requirements of roundwood for Northern Wood Preservers Inc.[a]

Product	Top Diameter Inside Bark (cm)	Log Length (m)
Veneer logs[b]	Min. 23 - Max. 51	2.69
Saw logs[c]	Min. 11 - No Max.	2.54 or 5.1
Pulp logs[d]	Min. 10 - Max. 41	2.54 only

[a] these represent spruce, pine (jack) and fir.

[b] extremely limited because of very high quality requirements and only spruce.

[c] in certain cases will go down to 10 cm.

[d] generally will accept logs below 10 cm min. diameter and down to 2.13 m long but these can not represent "too-high" a percentage of total mix; Fir (balsam) is generally not acceptable.

WOOD PRODUCT MIXES MODELLING

The basic dimensional requirements of wood products are top diameters (minimum and maximum diameters) and log length. To derive the log volume for any specific product from a tree, the lower and upper height limits along the bole are required. These can be estimated from the known minimum and maximum diameters using stem profile models.

Based on specific merchantability requirements for various wood products, the procedure for modelling the wood product mixes on a stand level is as following:

1. sort the stand information by species, dbh class and site class when available,
2. decide the types of products being produced from the stand and their corresponding dimensional requirements and species preference,
3. determine the priority of products, i.e. the order of various products to be generated from any given species,
4. estimate the total height for each species and dbh class from the dbh measurement using the corresponding regional height-diameter equation,
5. starting from the product with the highest priority, calculate the merchantable height limits for the various products from the known minimum or maximum top diameters using a stem profile equation,
6. calculate the volumes for various products for each species and dbh class; the basic steps at this stage

can be described as: a) calculate the volume for the first-order product for any given dbh class by species, b) continue on the calculation of the volumes for the second and third order products from the remaining stem based on the merchantability limits for these products, until the whole stem has been broken into different products, and

7. sum the volumes of the stand:

$$V = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \sum_{k=1}^{n_3} V_{ijk} \qquad [12]$$

Where:

V = Total volume of the stand,

n_1 = Number of species in the stand,

n_2 = Number of dbh classes of the i^{th} species,

n_3 = Number of wood product types to be modelled,

V_{ijk} = Volume of the k^{th} product type for the j^{th} dbh class of the i^{th} species.

The above procedure is illustrated in Figure 3. A Fortran program (Appendix D) has been written to facilitate the calculations for the above tasks.

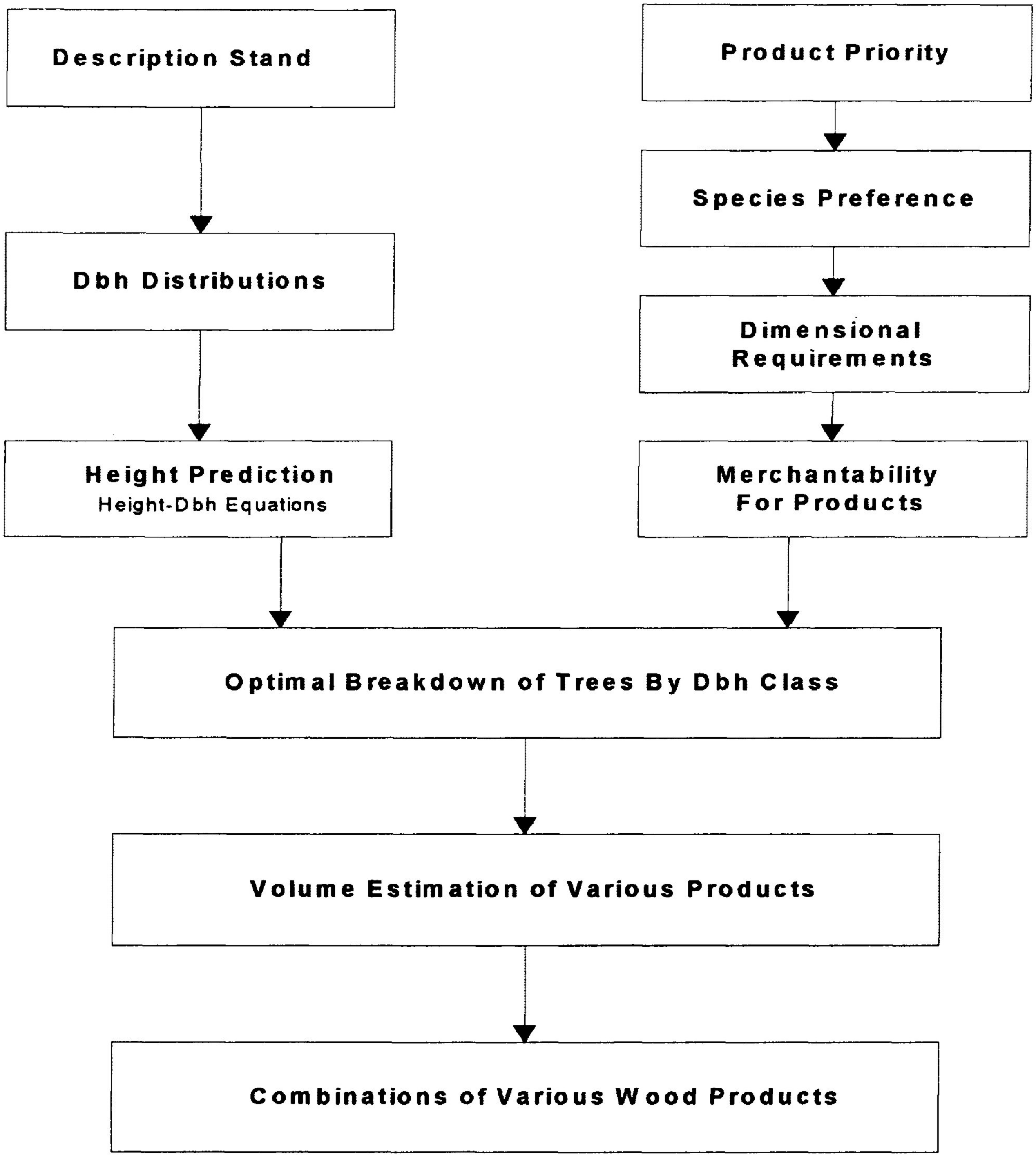


Figure 3. Flowchart for wood product model.

This wood product mix model can also be equally applied to individual trees. The basic elements in this system include:

1. stem profile models to estimate the height limits (H_U and H_L) and the segment volume for any specific product,
2. height prediction models used to estimate the total heights from dbh classes,
3. merchantability limits for wood products.

RESULTS AND DISCUSSION

SINGLE-ENTRY VOLUME TABLE CONSTRUCTION

Model Selection

Height-diameter functions

The comparison of several height prediction models is shown in Figure 4. The basal area function tends to flatten out after dbh reaches a certain point. As dbh becomes larger, the under-estimation of height by the basal area function gets greater. The quadratic function fitted the data set well within a certain range of dbh. It tends to drop down as dbh gets larger. For very large diameter classes, the under-estimation of height by the quadratic function is significant. In contrast, both the exponential equation and the Chapman-Richards function fitted the data sets very well, and the shapes are biologically reasonable. Table 7 lists the regression statistics (MSE and r^2) for comparing the exponential and Chapman-Richards models.

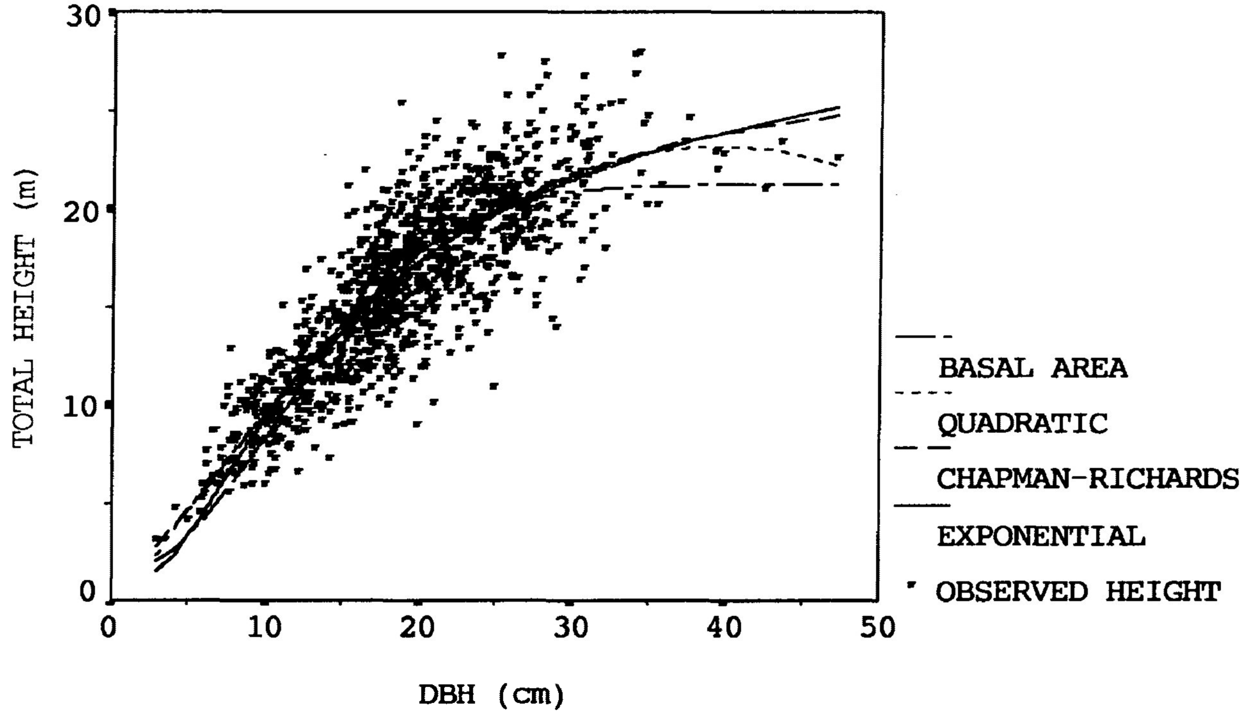


Figure 4. Comparison among several height prediction models for black spruce (site class X).

Table 7. Regression statistics for two height-dbh models

Species	Site Class	n	Exponential		Chapman-Richards	
			MSE (m)	r^2	MSE (m)	r^2
Black spruce	X	1207	5.59053	0.72248	5.49918	0.72724
	1	843	4.40240	0.62374	4.36049	0.62778
	2	368	3.52584	0.62822	3.49706	0.63227
	3	200	2.35056	0.52267	2.33494	0.52824
	4	67	1.94378	0.61505	1.92646	0.62435
Jack pine	X	126	7.69476	0.71712	7.24379	0.73584
	1	701	4.24996	0.67735	4.22817	0.67946
	2	728	2.82672	0.59836	2.82026	0.59983
	3	289	2.73158	0.55114	2.70972	0.55628
	4	211	2.76994	0.85248	2.70200	0.85679

For the majority of data points in the dbh range of 10 to 40 cm, both the exponential and the Chapman-Richards function yield almost the same predicted height (Figure 4). The differences between these two models only lay outside the 10 to 40 cm dbh range.

In terms of bias and mean absolute differences (Table 8), the Chapman-Richards function performed slightly better. However, it failed to produce good results for species with small-sample data, such as black ash and red maple. This is due to the fact that the Chapman-Richards function approaches the asymptote too quickly when the dependent variable is only weakly related to the independent variable (Huang *et al.*, 1992).

Table 8. Comparison between the exponential and the Chapman-Richards height prediction models

Species	Site Class	n	Model	$\bar{e}$ (m)	$\lvert\bar{e}\rvert$ (m)	SD (m)
Black spruce	X	1207	Exponential	.0202	1.8754	2.36
			Chapman-Richards	.0047	1.8523	2.34
	1	843	Exponential	.0109	1.6586	2.10
			Chapman-Richards	.0037	1.6433	2.09
	2	368	Exponential	.0029	1.4871	1.88
			Chapman-Richards	.0029	1.4795	1.86
	3	200	Exponential	-.0028	1.1882	1.53
			Chapman-Richards	-.0002	1.1779	1.52
	4	67	Exponential	-.0016	1.0304	1.38
			Chapman-Richards	.0029	1.0338	1.38
Jack pine	X	126	Exponential	.0431	2.2544	2.76
			Chapman-Richards	.0059	2.1782	2.67
	1	701	Exponential	.0059	1.6355	2.06
			Chapman-Richards	-.0005	1.6343	2.05
	2	728	Exponential	.0020	1.2952	1.68
			Chapman-Richards	-.0002	1.2902	1.68
	3	289	Exponential	-.0020	1.3055	1.65
			Chapman-Richards	.0003	1.3095	1.64
	4	211	Exponential	.0424	1.2766	1.66
			Chapman-Richards	-.0049	1.2474	1.64

For most of the data sets, the model assumptions of constant variance were approximately met by the exponential function. Figure 5 shows the plot of studentized residuals against the predicted height by the exponential function for jack pine (site class X). It shows approximately homogenous variance over the full range of predicted values.

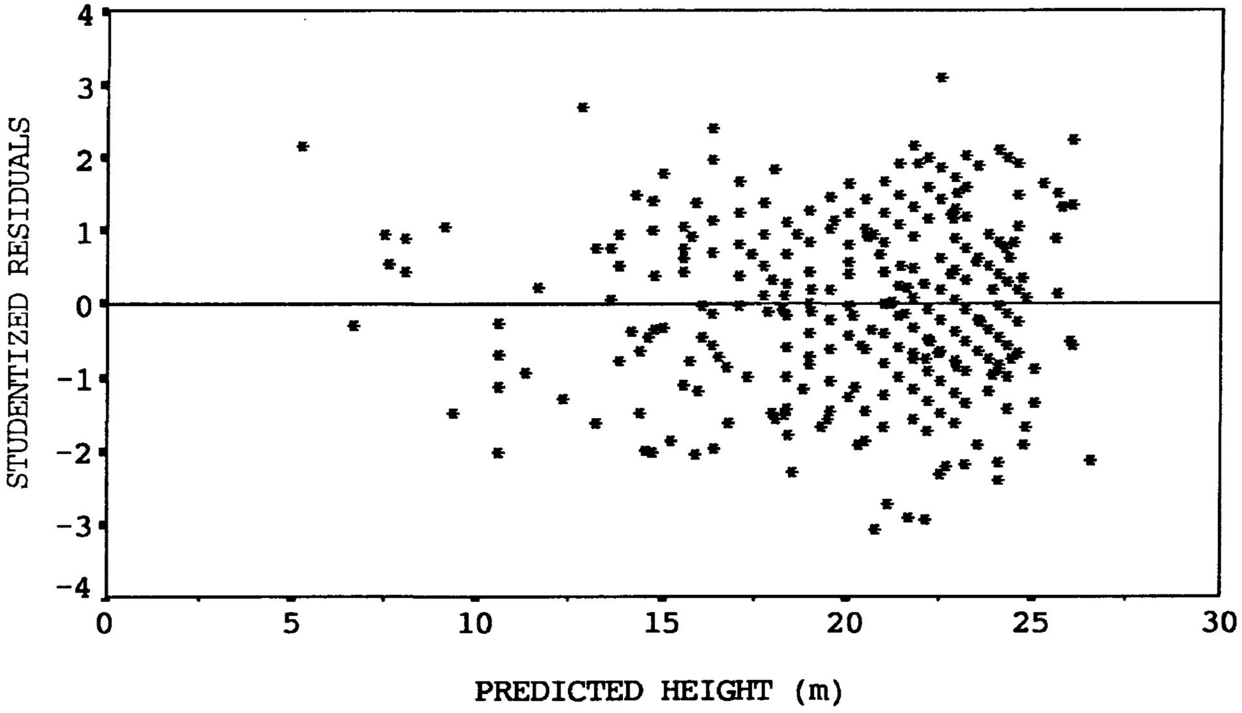


Figure 5. The plot of studentized residuals against the predicted height for jack pine (site class = X).

Age-height functions

The comparisons of the regression statistics (MSE and r^2) for the exponential function (Equation [5]) and the modified Weibull function (Equation [6]) are presented in Table 9. It is obvious that the modified Weibull function better results than the exponential-type function in fitting the data set (lower MSE and higher r^2).

Table 9. Regression statistics of models in predicting age

Species	Site Class	n	Exponential		Weibull	
			MSE (m)	r^2	MSE (m)	r^2
Black spruce	X	1199	184.68	.73337	160.55	.76888
	1	843	107.47	.87931	107.57	.87933
	2	368	147.64	.89497	147.98	.89502
	3	200	208.27	.81108	205.73	.81433
	4	67	438.70	.75556	440.31	.75844
Jack pine	X	125	36.71	.94106	35.39	.94365
	1	701	100.57	.85773	100.11	.85858
	2	727	135.07	.80911	134.76	.80936
	3	289	219.48	.75407	220.22	.75410
	4	211	114.45	.89324	114.58	.89363

The significant improvement of fit by the modified Weibull function over the exponential-type function is more evident from Figure 6, especially in the lower bound of total height. For trees of height under 10 m, the exponential-type function gave over-estimation for age, while the modified Weibull function worked well for the whole range of height.

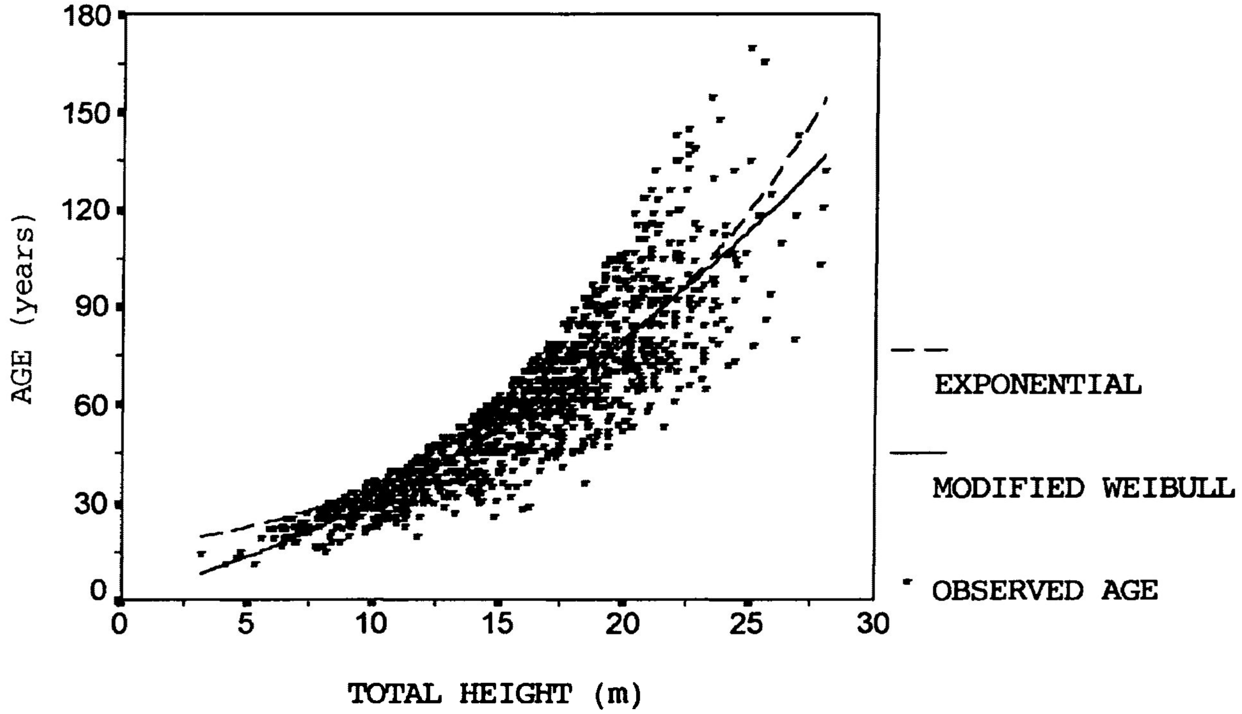


Figure 6. Plot of age prediction models for black spruce (site class X).

Accuracy of Single-Entry Volume Equations

Table 10 shows the paired samples t-test between the single-entry volumes and the stem analysis volumes. For all site classes of jack pine and black spruce, except for black spruce site class 3, the single-entry volume equations fitted the stem analysis data well with relative bias less than 10 percent. For jack pine, the differences between the single-entry volumes and the stem analysis were insignificant for site class X, 3, 4. As expected, the relative differences between the single-entry and standard volumes are small for most cases (Table 11), which demonstrated the consistency between the single-entry and standard volume equations. These comparisons are also illustrated in Figures 7 and 8. Table 12 lists the comparisons of the standard volumes and the stem analysis volumes.

Table 10. Comparisons between single-entry and stem analysis volumes

Species	Site Class	n	Bias (m^3)	Mean Volume (m^3)	Relative Bias (%)	Std. Err. (m^3)	t	2-tail Sig.
Jack Pine	X	120	0.0007	0.0156	4.49	0.001	0.95	0.345
	1	238	0.0062	0.1802	3.44	0.003	2.12	0.035
	2	172	-0.0098	0.1401	-7.00	0.003	-3.71	0.000
	3	156	-0.0042	0.1290	-3.26	0.002	-1.92	0.056
	4	84	-0.0024	0.0278	-8.63	0.002	-1.47	0.146
Black Spruce	X	175	0.0099	0.1666	5.94	0.002	5.44	0.000
	1	312	0.0021	0.0915	2.30	0.001	2.32	0.021
	2	210	-0.0019	0.0567	-3.35	0.001	-3.02	0.003
	3	83	-0.0024	0.0210	-11.43	0.000	-9.05	0.000
	4	37	-0.0023	0.0236	-9.75	0.000	-5.68	0.000

Table 11. Comparisons between single-entry volumes and standard volumes

Species	Site Class	n	Bias (m^3)	Mean Volume (m^3)	Relative Bias (%)	Std. Err. (m^3)	t	2-tail Sig.
Jack Pine	X	120	-0.0001	0.0148	-0.68	0.000	-0.19	0.846
	1	238	-0.0037	0.1703	-2.17	0.002	-1.89	0.067
	2	172	-0.0113	0.1387	-8.15	0.003	-5.45	0.000
	3	156	-0.0079	0.1254	-6.30	0.001	-5.88	0.000
	4	84	-0.0004	0.0297	-1.35	0.001	-0.48	0.630
Black Spruce	X	175	0.0066	0.1633	4.04	0.001	6.77	0.000
	1	312	0.0023	0.0917	2.51	0.001	4.07	0.000
	2	210	0	0.0586	0	0.000	-0.04	0.969
	3	83	-0.0014	0.0220	-6.36	0.000	-5.37	0.000
	4	37	0	0.0259	0	0.000	-0.06	0.952

Table 12. Comparisons between standard volumes and stem analysis volumes

Species	Site Class	n	Bias (m^3)	Mean Volume (m^3)	Relative Bias (%)	Std. Err. (m^3)	t	2-tail Sig.
Jack Pine	X	120	0.0008	0.0156	5.13	0.000	1.78	0.078
	1	238	0.0099	0.1802	5.49	0.001	7.28	0.000
	2	172	0.0015	0.1401	1.07	0.001	1.41	0.162
	3	156	0.0036	0.1290	2.79	0.001	2.64	0.009
	4	84	-0.0020	0.0278	-7.19	0.001	-1.66	0.101
Black Spruce	X	175	0.0033	0.1666	1.98	0.001	2.36	0.019
	1	312	-0.0002	0.0915	-0.22	0.001	0.42	0.675
	2	210	-0.0019	0.0567	-3.35	0.000	-5.56	0.000
	3	83	-0.0011	0.0210	-5.24	0.000	-6.72	0.000
	4	37	-0.0023	0.0236	-9.75	0.000	-5.93	0.000

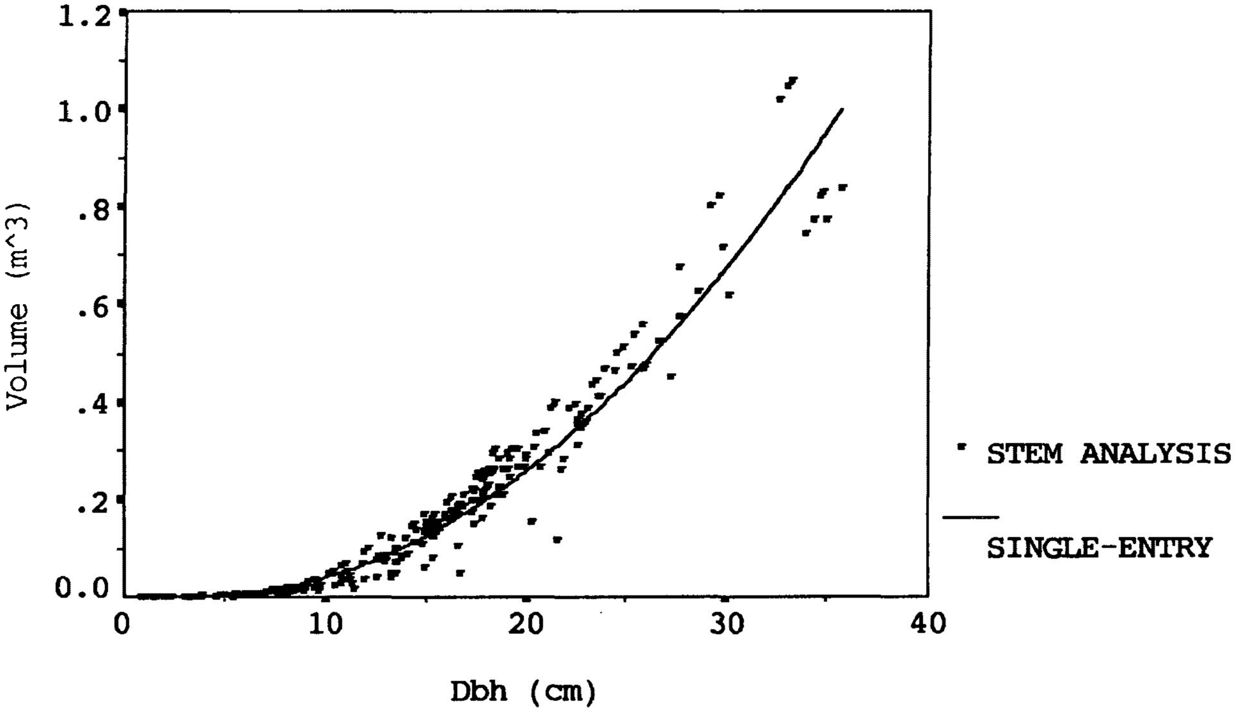


Figure 7. Scatterplot of single-entry volumes versus stem analysis volumes for jack pine (site class 1).

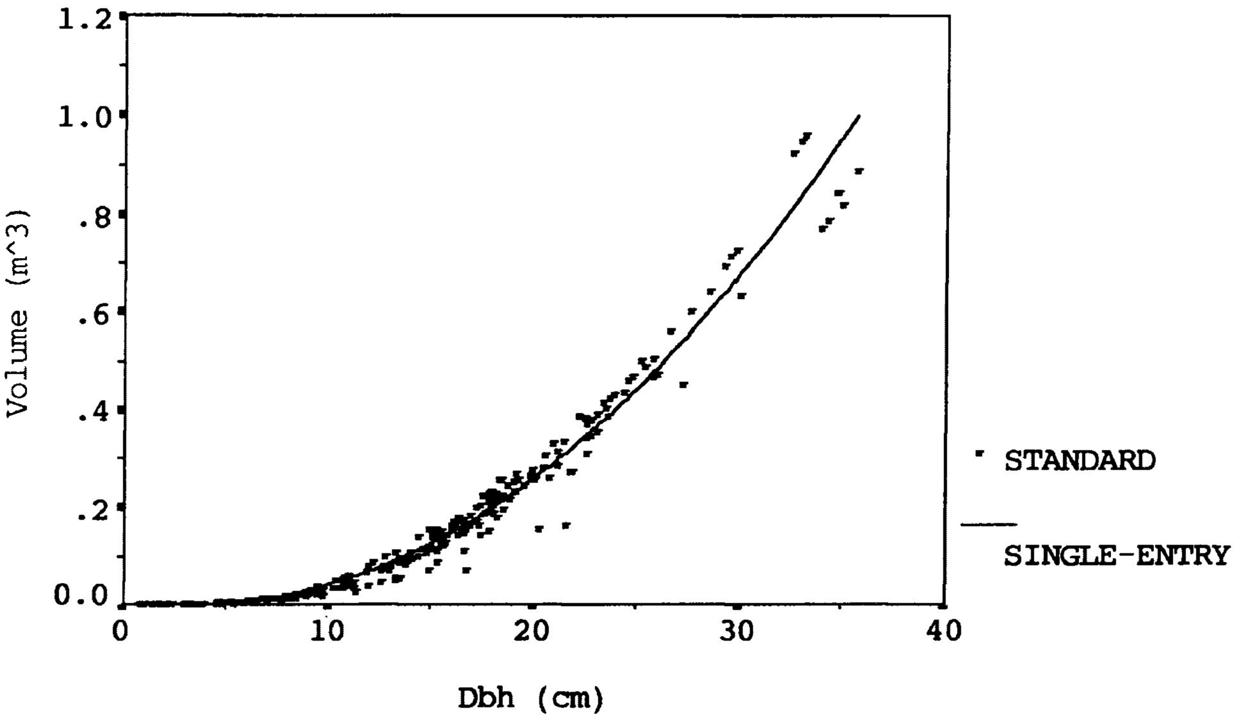


Figure 8. Scatterplot of single-entry volumes versus standard volumes for jack pine (site class 1).

Two Approaches of Deriving Height Information

Zakrzewski (1993) proposed an approach of deriving height using a central tree method. Maurer's (1993) approach, on the other hand, is referred to as a site stratification method.

For the site stratification method, the height-dbh equations were derived by site class, hence, there is no direct comparison to the central tree method which generates the equations by species. By employing the weighted average criteria for all the site classes, however, it is easier to compare these two approaches. From Table 13, it is apparent that the site stratification method yielded better results (lower MSE and higher r^2).

Table 13. Comparisons of the two height derivation approaches

Species	Criteria	Site Stratification Method						Central Tree Method
		scX	sc1	sc2	sc3	sc4	All	
Jack pine	n	126	701	728	289	211		1903
	MSE	7.695	4.250	2.827	2.732	2.770	3.592	5.710
	r^2	0.717	0.677	0.598	0.551	0.852	0.652	0.571
Aspen	n	48	304	572	476	82		1386
	MSE	6.561	6.092	4.113	4.165	4.712	4.648	9.405
	r^2	0.865	0.758	0.783	0.743	0.599	0.757	0.620
Black spruce	n	1207	843	368	200	67		2687
	MSE	5.591	4.402	3.526	2.351	1.944	4.602	5.801
	r^2	0.722	0.624	0.628	0.523	0.615	0.661	0.658

Comparative Analysis of Regional and Provincial LVT's

The two-tailed t-test statistics (Table 14) show that, for jack pine, coefficients b_1 and b_2 in Equation [1] between the two regions (Northwest and Northeast) were significantly different except b_2 of site class 3. This result justified the stratification of the single-entry volume equations for jack pine by region. The height-diameter equations for black spruce site classes 1, 2, 3 and 4 show overall insignificant differences between the two regions. Only site class X tested significantly different for both b_1 and b_2 between the two regions (Table 14).

Table 14. The statistics of 2-tail t-tests for testing the regression coefficients differences for the regional models

Species	Site Class	df	t	
			b_1	b_2
Jack pine	X	382	3.449**	4.329**
	1	874	6.999**	9.707**
	2	495	10.442**	7.056**
	3	176	3.401**	0.926
	4	109	3.965**	3.103**
Black spruce	X	1108	6.111**	4.178**
	1	741	1.555	2.213*
	2	371	0.838	0.044
	3	102	1.130	0.030
	4	45	2.287*	1.360

* Significant at 95% confidence level
** Significant at 99% confidence level

The provincial single-entry volume equations were constructed in the same way as the regional equations, by combining the data sets from the regions across the province. A further attempt was made to compare the provincial equations to the regional equations to determine the applicability of the models on a province-wide basis.

For black spruce, the provincial single-entry volume equations gave very close volume estimates as those by the regional equations for both regions. The relative differences between the provincial and the corresponding regional estimates for all the site classes were less than 5 percent (Table 15).

When the provincial volume equations for jack pine were used to the Northeastern region, the differences were apparent, especially between the provincial equations and the Northeastern's equations (Table 15).

Table 15. Comparisons of regional and provincial LVT's

Species	Site Class	Region[a]	n	Mean Vol. (m^3)	Mean Diff.[b] (m^3)	Relative Diff.(%)
Jack pine	X	NE	643	.5245	.0005	0.10
		NW	126	.3923	.0013	0.33
	1	NE	1048	.4622	-.0107	-2.32
		NW	701	.4356	.0079	1.81
	2	NE	267	.3869	-.0235	-6.07
		NW	727	.3343	.0023	0.69
	3	NE	68	.2809	-.0233	-8.29
		NW	289	.2681	.0033	1.23
	4	NE	12	.0998	-.0123	-12.32
		NW	56	.1456	-.0043	-2.94
Black spruce	X	NE	1013	.2525	-.0046	-1.82
		NW	1207	.2415	.0025	1.04
	1	NE	643	.2002	-.0006	-0.30
		NW	843	.1714	.0002	0.12
	2	NE	378	.1418	-.0009	-0.63
		NW	368	.1380	.0017	1.23
	3	NE	109	.0859	-.0016	-1.86
		NW	200	.1051	.0017	1.62
	4	NE	27	.0641	-.0031	-4.84
		NW	67	.0613	.0009	1.47

[a] NE = Northeastern Region; NW = Northwestern Region,

[b] Defined as the volume predicted by the regional equation minus the volume predicted by the provincial equation.

STEM PROFILE MODELLING

Comparisons Using The Sample Data

The summary statistics of the model performance for diameter, height and volume predictions for the sample data are displayed in Table 16. To make it easier to compare these models, all the criteria for the 12 species tested were combined and recomputed using weighted average of the values for each model. The biases were calculated by ignoring the sign.

The bias in predicting diameters ranged from .1235 cm for Demaerschalk's model to .3278 cm for the model by Kozak *et al.*, while mean absolute difference in predicting diameters ranged from .8617 cm for Max and Burkhart's model to 1.0782 cm for the model by Kozak *et al.* In predicting heights, Max and Burkhart's model did the best job for both bias and the mean absolute difference (.0865 and .6852 m, respectively), followed by Demaerschalk's model (.1174 and .7390 m, respectively). With volume prediction, Demaerschalk's model performed the best for both bias and the mean absolute difference (.0206 and .0485 m^3, respectively), followed by Ormerod's model (.0273 and .0533 $m^{3,}$ respectively), then by Max and Burkhart's model (.0383 and

.0601 m^3, respectively). The model by Kozak *et al.* ranked the lowest.

Table 16. Summary of bias, mean absolute difference, and standard deviation of several stem profile models for the sample data

Model	Bias[a]	MAD[b]	SD[c]
<u>Diameter Prediction (cm)</u>			
Demaerschalk (1972)	.1235	.9786	1.4693
Max and Burkhart (1976)	.1973	.8617	1.3380
Kozak *et al.* (1969)	.3278	1.0782	1.5802
Ormerod (1973)	.2449	1.0675	1.5328
<u>Height Prediction (m)</u>			
Demaerschalk (1972)	.1174	.7390	1.0475
Max and Burkhart (1976)	.0865	.6852	1.0290
Kozak *et al.* (1969)	.2041	.8070	1.1271
Ormerod (1973)	.2198	.8269	1.1330
<u>Volume Prediction (m^3)</u>			
Demaerschalk (1972)	.0206	.0485	.0863
Max and Burkhart (1976)	.0383	.0601	.1158
Kozak *et al.* (1969)	.0499	.0670	.1260
Ormerod (1973)	.0273	.0533	.0988

[a] The bias is defined as the measured values minus the predicted values. For volume prediction, stem analysis volumes (see MacLeod 1978, Appendix 1) were taken to be the true values.

[b] Mean absolute differences of bias.

[c] Standard deviation of bias.

Further comparisons were made among the models to evaluate their ability to predict diameters, heights and volumes in relation to relative height along the tree bole. The results are presented in Tables 17 and 18. Several trends were revealed:

(1) It is apparent that the bias for both diameter and height predictions for Max and Burkhart's model are significantly smaller than those for the other models for both species at most positions, especially at the lower portions of the bole. This again indicates the superior predictive abilities of Max and Burkhart's model in predicting top diameters and section heights.

(2) In general, Max and Burkhart's model over-predicted (negative bias) diameters and volumes along the entire bole but under-estimated (positive bias) heights for jack pine at the middle positions. The other three models over-predicted diameters, heights and volumes in the lower and upper bole, but under-estimated in the middle positions

(3) There was no apparent differences among the four models in predicting the section volumes.

Table 17. Summary of error of stem profile models by relative height class for jack pine

Section of relative height	n	Diameter (cm)		Height (m)		Volume (m^3)	
		Bias	SD	Bias	SD	Bias	SD
<u>Demaerschalk (1972)</u>							
$0.0 \le x < 0.1$	525	.372	1.543	-.449	.845	-.0025	.0087
$0.1 \le x < 0.2$	272	-.709	.778	-.973	1.000	-.0045	.0049
$0.2 \le x < 0.3$	267	-.663	1.075	-.797	1.196	-.0053	.0096
$0.3 \le x < 0.4$	272	-.408	1.642	-.373	1.441	-.0041	.0110
$0.4 \le x < 0.5$	264	-.042	1.106	.059	1.106	-.0022	.0099
$0.5 \le x < 0.6$	268	.221	.944	.356	.956	-.0003	.0077
$0.6 \le x < 0.7$	267	.389	1.090	.525	.996	.0008	.0063
$0.7 \le x < 0.8$	264	.413	1.241	.532	1.129	.0012	.0059
$0.8 \le x < 0.9$	265	-.164	1.363	.020	.870	.0004	.0050
$0.9 \le x \le 1.0$	521	-.656	1.043	-.233	.487	-.0009	.0033
All	3185	-.129	1.307	-.169	1.088	-.0017	.0077
<u>Max and Burkhart (1976)</u>							
$0.0 \le x < 0.1$	525	-.082	1.001	-.152	.484	-.0046	.0012
$0.1 \le x < 0.2$	272	-.170	.814	-.223	1.296	-.0009	.0044
$0.2 \le x < 0.3$	267	-.396	1.172	-.427	1.588	-.0033	.0099
$0.3 \le x < 0.4$	272	-.379	1.737	-.285	1.722	-.0039	.0124
$0.4 \le x < 0.5$	264	-.227	1.257	-.095	1.347	-.0033	.0119
$0.5 \le x < 0.6$	268	-.114	1.108	.055	1.132	-.0022	.0101
$0.6 \le x < 0.7$	267	-.039	1.231	.150	1.062	-.0015	.0085
$0.7 \le x < 0.8$	264	.006	1.349	.201	1.141	-.0009	.0075
$0.8 \le x < 0.9$	265	-.332	1.428	-.065	.765	-.0009	.0060
$0.9 \le x \le 1.0$	521	-.501	.968	-.151	.448	-.0007	.0031
All	3185	-.234	1.206	-.103	1.102	-.0023	.0090
<u>Kozak *et al.* (1969)</u>							
$0.0 \le x < 0.1$	525	.172	1.449	-.501	.861	-.0033	.0093
$0.1 \le x < 0.2$	272	-.779	.966	-.949	1.097	-.0063	.0086
$0.2 \le x < 0.3$	267	-.672	1.256	-.718	1.316	-.0064	.0127
$0.3 \le x < 0.4$	272	-.377	1.744	-.271	1.534	-.0048	.0132
$0.4 \le x < 0.5$	264	-.007	1.218	.153	1.202	-.0026	.0115
$0.5 \le x < 0.6$	268	.239	1.037	.432	1.058	-.0006	.0090
$0.6 \le x < 0.7$	267	.349	1.163	.541	1.085	.0004	.0073
$0.7 \le x < 0.8$	264	.280	1.305	.466	1.214	.0006	.0066
$0.8 \le x < 0.9$	265	-.428	1.447	-.162	.928	-.0005	.0057
$0.9 \le x \le 1.0$	521	-.924	1.225	-.335	.554	-.0018	.0038
All	3185	-.242	1.388	-.182	1.157	-.0025	.0092
<u>Ormerod (1973)</u>							
$0.0 \le x < 0.1$	525	-.422	1.435	-.976	1.081	-.0048	.0095
$0.1 \le x < 0.2$	272	-1.286	1.000	-1.539	.981	-.0100	.0108
$0.2 \le x < 0.3$	267	-1.084	1.281	-1.177	1.171	-.0093	.0143
$0.3 \le x < 0.4$	272	-.675	1.746	-.611	1.394	-.0068	.0141
$0.4 \le x < 0.5$	264	-.157	1.204	-.041	1.059	-.0037	.0118
$0.5 \le x < 0.6$	268	.255	1.005	.389	.925	-.0008	.0087
$0.6 \le x < 0.7$	267	.565	1.124	.677	.968	.0011	.0067
$0.7 \le x < 0.8$	264	.718	1.255	.784	1.107	.0021	.0059
$0.8 \le x < 0.9$	265	.245	1.352	.328	.894	.0015	.0048
$0.9 \le x \le 1.0$	521	-.401	.923	-.119	.468	-.0001	.0032
All	3185	-.258	1.379	-.284	1.241	-.0030	.0102

Table 18. Summary of error of stem profile models by relative height class for black spruce

Section of relative height	n	Diameter (cm)		Height (m)		Volume (m^3)	
		Bias	SD	Bias	SD	Bias	SD
		Demaerschalk (1972)					
0.0 ≤ x < 0.1	142	.210	1.505	-.550	.908	-.0008	.0026
0.1 ≤ x < 0.2	89	-.769	.512	-.951	.575	-.0029	.0031
0.2 ≤ x < 0.3	90	-.387	.505	-.448	.582	-.0021	.0024
0.3 ≤ x < 0.4	95	-.007	.475	.015	.543	-.0007	.0016
0.4 ≤ x < 0.5	94	.138	.562	.192	.717	.0002	.0015
0.5 ≤ x < 0.6	89	.307	.629	.387	.669	.0006	.0016
0.6 ≤ x < 0.7	87	.348	.660	.415	.662	.0008	.0006
0.7 ≤ x < 0.8	88	.241	.727	.295	.658	.0006	.0015
0.8 ≤ x < 0.9	88	.074	.654	.121	.505	.0003	.0012
0.9 ≤ x ≤ 1.0	169	-.106	.464	-.013	.266	.0001	.0006
All	1031	.006	.829	-.076	.740	-.0004	.0022
		Max and Burkhart (1976)					
0.0 ≤ x < 0.1	142	-.053	.830	-.169	.455	-.0013	.0033
0.1 ≤ x < 0.2	89	-.097	.496	-.175	.875	0	.0019
0.2 ≤ x < 0.3	90	-.096	.525	-.107	.899	-.0007	.0018
0.3 ≤ x < 0.4	95	-.023	.506	.018	.765	-.0005	.0016
0.4 ≤ x < 0.5	94	-.125	.590	-.108	.750	.0004	.0016
0.5 ≤ x < 0.6	89	-.134	.645	-.075	.684	-.0005	.0018
0.6 ≤ x < 0.7	87	-.136	.688	-.051	.600	-.0005	.0018
0.7 ≤ x < 0.8	88	-.064	.731	.006	.530	-.0013	.0016
0.8 ≤ x < 0.9	88	-.024	.643	.025	.443	0	.0012
0.9 ≤ x ≤ 1.0	169	-.175	.475	-.036	.267	0	.0006
All	1031	-.097	.623	-.070	.628	-.0004	.0019
		Kozak *et al.* (1969)					
0.0 ≤ x < 0.1	142	.053	1.507	-.615	.998	-.0012	.0024
0.1 ≤ x < 0.2	89	-.783	.625	-.937	.705	-.0033	.0038
0.2 ≤ x < 0.3	90	-.353	.573	-.405	.665	-.0023	.0027
0.3 ≤ x < 0.4	95	.049	.493	.075	.571	-.0007	.0018
0.4 ≤ x < 0.5	94	.177	.556	.232	.630	.0001	.0015
0.5 ≤ x < 0.6	89	.287	.604	.371	.671	.0004	.0016
0.6 ≤ x < 0.7	87	.236	.644	.317	.680	.0005	.0015
0.7 ≤ x < 0.8	88	.009	.719	.082	.678	.0002	.0014
0.8 ≤ x < 0.9	88	-.330	.680	-.223	.512	-.0002	.0013
0.9 ≤ x ≤ 1.0	169	-.461	.570	-.196	.312	-.0003	.0007
All	1031	-.129	.855	-.158	.768	-.0007	.0023
		Ormerod (1973)					
0.0 ≤ x < 0.1	142	.235	1.522	-.543	.883	-.0007	.0027
0.1 ≤ x < 0.2	89	-.756	.463	-.943	.536	-.0028	.0029
0.2 ≤ x < 0.3	90	-.385	.474	-.452	.560	-.0020	.0022
0.3 ≤ x < 0.4	95	-.025	.474	-.005	.543	-.0006	.0015
0.4 ≤ x < 0.5	94	.126	.573	.177	.627	.0002	.0015
0.5 ≤ x < 0.6	89	.297	.647	.380	.680	.0006	.0016
0.6 ≤ x < 0.7	87	.345	.666	.412	.665	.0008	.0016
0.7 ≤ x < 0.8	88	.236	.725	.292	.660	.0006	.0014
0.8 ≤ x < 0.9	88	.074	.651	.121	.505	.0003	.0012
0.9 ≤ x ≤ 1.0	169	-.107	.461	-.013	.266	.0001	.0006
All	1031	.006	.830	-.079	.731	-.0003	.0021

Comparisons Using The Independent Data

The preceding results described the fit of the four stem profile models to the data used to build the models. For the stem profile models tested, independent stem analysis data sets were also used to validate the model performance to determine their applicability beyond the sample data. Two sources of independent data sets were used: (1) the reserved FMI data, and (2) the stem analysis data from Central Ontario.

As in the tests for the sample data, Max and Burkhart's model achieved better results than the other models in predicting diameters and heights. Moreover, Max and Burkhart's model was the best for volume prediction for the independent data. These results are shown in Table 19.

Table 19. Summary of bias, mean absolute difference, and standard deviation of several stem profile models for the independent data

Model	Bias	MAD	SD
Diameter Prediction (cm)			
Demaerschalk (1972)	.3265	1.5007	2.0366
Max and Burkhart (1976)	.2789	1.3584	1.9075
Kozak *et al.* (1969)	.4600	1.6117	2.1507
Ormerod (1973)	.2749	1.6372	2.1716
Height Prediction (m)			
Demaerschalk (1972)	.2505	1.0990	1.4528
Max and Burkhart (1976)	.2295	1.0187	1.4322
Kozak *et al.* (1969)	.2905	1.1791	1.5468
Ormerod (1973)	.2295	1.2160	1.5835
Volume Prediction* (m^3)			
Demaerschalk (1972)	.0046	.0092	.0162
Max and Burkhart (1976)	.0038	.0090	.0170
Kozak *et al.* (1969)	.0052	.0098	.0179
Ormerod (1972)	.0074	.0104	.0172

* Stem analysis data in Northwestern Ontario.

The ranking results (Table 20) show that Max and Burkhart's model ranked the highest in ability to predict top diameters and heights. Demaerschalk's model ranked second, followed by Ormerod's model and the model by Kozak *et al.* With volume prediction, however, Demaerschalk's model ranked the highest, followed closely by Max and Burkhart's model, then by Kozak *et al.*'s model. Ormerod's model gave the poorest results for volume prediction.

Table 20. Overall rankings of the four models for diameter, height and volume prediction

Model	Test data	Ranking*				Overall ranking
		Bias	MAD	SD	Sum	
		Diameter				
Demaerschalk	sample	22	27	26	75	
(1972)	independent	26	27	25	78	
	All	48	54	51	155	2
Max and Burkhart	sample	27	12	13	52	
(1976)	independent	21	11	10	42	
	All	48	23	23	94	1
Kozak *et al.*	sample	39	42	44	125	
(1969)	independent	26	33	37	96	
	All	65	75	81	221	4
Ormerod (1973)	sample	32	39	37	108	
	independent	27	31	28	86	
	All	59	70	65	194	3
		Height				
Demaerschalk	sample	26	29	27	82	
(1972)	independent	26	24	23	73	
	All	52	51	50	155	2
Max and Burkhart	sample	17	12	16	45	
(1976)	independent	25	20	18	63	
	All	42	32	36	108	1
Kozak *et al.*	sample	40	41	41	122	
(1969)	independent	19	28	34	81	
	All	59	69	75	203	4
Ormerod (1973)	sample	37	38	36	111	
	independent	30	28	25	83	
	All	67	66	61	194	3
		volume				
Demaerschalk	sample	22	21	19	62	
(1972)	independent	17	16	16	49	
	All	39	37	35	111	1
Max and Burkhart	sample	31	28	31	90	
(1976)	independent	10	10	9	29	
	All	41	38	40	119	2
Kozak *et al.*	sample	36	37	39	112	
(1969)	independent	7	7	7	22	
	All	43	44	45	134	3
Ormerod (1973)	sample	31	34	31	96	
	independent	18	17	17	52	
	All	49	51	49	148	4

* All species combined.

A test to compare Demaerschalk's model and Max and Burkhart's model to the standard volume equations (Honer *et al.*, 1983) was conducted. The results are shown in Table 21. It is apparent that, for most species, both models gave satisfactory results. For black spruce, balsam fir, white birch and white cedar, however, the biases are relative large. Another trend shown in Table 21 is that the over-estimation becomes greater as the dbh class gets larger. Generally, Demaerschalk's model performed better in volume prediction for softwood species, while the model by Max and Burkhart demonstrated superior results for hardwood species.

Table 21. Comparison of stem profile models to single-entry volume equations for volume prediction

Species	Diameter Class (cm)	Mean Volume (m^3)	Demaerschalk		Max and Burkhart	
			Mean Bias (m^3)	%	Mean Bias (m^3)	%
Largetooth aspen	Total	1.0236	.0378	3.69	.0379	3.70
	1- 20	.0816	.0007	.86	.0017	2.08
	21- 40	.7520	.0179	2.38	.0251	3.34
	41- 60	2.2372	.0947	4.23	.0870	3.89
Aspen (gen.)	Total	1.2077	-.0278	-2.30	.0072	.60
	1- 20	.0843	-.0016	-1.90	-.0017	-2.02
	21- 40	.8646	-.0196	-2.32	-.0002	-.02
	41- 60	2.6742	-.0621	-2.32	.0235	.88
Trembling aspen	Total	1.0837	-.0398	-3.67	.0081	.75
	1- 20	.0801	-.0024	-3.00	-.0011	-1.37
	21- 40	.7866	-.0213	-2.71	.0020	.25
	41- 60	2.3843	-.0956	-4.01	.0234	.98
Balsam fir	Total	.7859	-.1099	-13.98	-.1144	-14.56
	1- 20	.0716	-.0037	-5.17	-.0034	-4.75
	21- 40	.5931	-.0692	-10.61	-.0741	-12.49
	41- 60	1.6930	-.2576	-15.22	-.2657	-15.69
White birch	Total	.7845	-.1070	-13.64	-.0662	-8.44
	1- 20	.0758	-.0016	-2.11	-.0004	-.52
	21- 40	.5975	-.0646	-10.81	-.0412	-6.90
	41- 60	1.6801	-.2548	-15.17	-.1571	-9.35
White cedar	Total	.5932	-.0704	-11.87	-.0430	-7.25
	1- 20	.0544	.0053	9.74	.0049	9.01
	21- 40	.4486	-.0225	-5.02	-.0147	-3.28
	41- 60	1.2767	-.1835	-14.37	-.1095	-8.58
Jack pine	Total	.9768	-.0279	-2.86	-.0742	-7.60
	1- 20	.0815	-.0043	-5.28	-.0018	-2.21
	21- 40	.7252	-.0282	-3.89	-.0465	-6.41
	41- 60	2.1237	-.0512	-2.41	-.1744	-8.21
Red pine	Total	.9768	.0181	1.85	-.0538	5.51
	1- 20	.0716	-.0031	-4.33	-.0002	-.28
	21- 40	.7080	-.0004	-.06	-.0299	-4.22
	41- 60	2.1508	.0579	2.69	-.1313	-6.10
White pine	Total	1.0854	-.0178	-1.64	-.0754	-6.95
	1- 20	.0702	-.0019	-2.71	0	0
	21- 40	.7687	-.0139	-1.81	-.0392	-5.10
	41- 60	2.4173	-.0377	-1.56	-.1870	-7.74
Black spruce	Total	.8742	-.1343	-15.36	-.1512	-17.30
	1- 20	.0734	-.0060	-8.17	-.0054	-7.36
	21- 40	.6505	-.0861	-13.24	-.0980	-15.07
	41- 60	1.8987	-.3109	-16.37	-.3503	-18.45
White spruce	Total	.6938	-.0057	-.82	-.0209	-3.01
	1- 20	.0669	-.0004	-.60	-.0005	-.75
	21- 40	.5278	-.0037	-.70	-.0118	-2.24
	41- 60	1.4868	-.0130	-.87	-.0503	-3.38

WOOD PRODUCT MIXES MODELLING

The wood product mix model was developed to estimate volumes for various wood products generated from individual stands or trees. Obviously, the wood product mix model does not depend on any specific criteria. The following example demonstrates the application of the model. The stand information (Table 22) was taken from stand "Kelvin 2656" (Maurer, 1994).

Table 22. Diameter distributions of input stand

Dbh Class (cm)	Stems Per Hectare					
	Trembling aspen	Balsam fir	White birch	Jack pine	Black spruce	White spruce
6		141				
8		96	32		16	16
10		51	41		10	
12		35	14		35	
14		42	21		26	
16	12	24	16		4	8
18	9	13	3	13	9	3
20	18	5	8	8	3	3
22	25	8		8	2	6
24	21	4	2	9		7
26	29			9	3	3
28	27			16	1	1
30	10	1		9		1
32	13			3	1	2
34	15			1		
36	7			3		2
38	1			4		
40	4					
42	3			1		
44	4					
46	2					
56	1					

The results of wood product mixes modelled are presented in Table 23. These volumes were generated from Demaerschalk' (1972) stem profile model. Table 24 lists the comparisons between the volumes and those obtained from the standard volume equations (Honer *et al.*, 1983). It is apparent, in terms of volume estimation, that the wood product mix model was consistent to the volume equation approach.

Table 23. Output of stand modelling*

Spp.	Ave. Dbh (cm)	Ave. Height (m)	Ave. Volume (m^3)	n	Volumes For Products (m^3) Veneer	Sawlog#	Pulp#	Chips
AT	26.89	21.84	.6266	201	.00	.00	.00	125.95
BF	9.89	8.27	.0501	420	.00	10.10	.00	10.95
BW	12.01	12.36	.0764	137	.00	.00	.00	10.46
PJ	25.83	19.73	.5158	84	.00	.00	41.04	2.29
SB	13.45	11.89	.1049	110	1.22	.00	7.07	3.25
SW	18.12	12.81	.2242	52	3.57	.00	7.06	1.03
ALL	15.73	13.14	.2231	1004	4.79	10.10	55.17	153.93

* This table only shows the summary information for the species and the stand. The merchantability limits are taken from Table 6.

\# Because of the similar dimensional requirements, these two products can be assigned either way depending on the preference of the products.

Table 24. Volume comparisons between wood product mix model and standard volume equations

Species	Volume Estimates (m^3) Product model	Honer *et al.*	Bias	%
Trembling aspen	125.95	122.63	3.32	2.64
Balsam fir	21.05	20.33	0.72	3.42
White birch	10.46	10.52	-0.06	-0.57
Jack pine	43.33	41.70	1.66	3.83
Black spruce	11.54	10.66	0.84	7.28
White spruce	11.66	11.53	0.13	1.11
All	223.99	217.37	6.62	2.96

CONCLUSIONS AND RECOMMENDATIONS

CONCLUSIONS

In deriving the height variable for the construction of single-entry volume tables, the site stratification method demonstrated superior performance over the central tree method. The use of a site stratified height-dbh equation greatly reduced the variation of height prediction within a region, thus leading to a wider application for the geographic area in which the equation was derived. For black spruce in the two northern Ontario regions, the provincial single-entry volume tables will apply nearly equally well as the regional tables. Generally, there is a wider variation in the height-dbh relationship between the two regions for jack pine and the other species tested (Appendix A). For these species, the applications of the single-entry volume equations on a province-wide basis cannot be justified.

The exponential height function of Height = $1.3 + b_1 \times \exp(b_2/Dbh)$, as proved by many others (Maurer, 1993; Arabatzis and Burkhart, 1992), produced satisfactory height estimates for most tree species tested in this study. Another height-diameter function deserving some attention is the Chapman-Richards function suggested by Huang *et al.* (1992). This function can be very useful when fitted to large sample data sets. Unfortunately, it failed to fit the height-diameter relationship well for some species with small sample data sets.

In predicting age, the modified Weibull function of Age $= b_1 \times \exp(b_2 \times \text{Height}^{b_3})$ showed a significant improvement over the exponential-type function of Age $= 1.3 + b_1 \times \exp(b_2 \times \text{Height})$. It produced reasonable age estimates for the whole range of height class observed in this study.

The wood product mix model, based on the stem profile equations and dbh distributions, offers an efficient way of estimating merchantable volumes by wood products from merchantable stands. It is useful in evaluating alternative uses of timber resources. The wood product mix model can be applied to any merchantable stands in accordance with varying product demands and dimensional requirements. The model itself is flexible, and capable of estimating various wood products, regardless of the specific criteria for a given product. With the Fortran program written to facilitate the calculations, the model is easy and straightforward to apply.

Of the stem profile models tested, the segmented polynomial model by Max and Burkhart (1976) proved to be the most accurate and precise in predicting top diameters and section heights, especially at the lower portions of the tree bole, while the model derived from the logarithmic volume equation by Demaerschalk (1972) ranked the highest overall for volume prediction, followed closely by Max and Burkhart's model. In general, Demaerschalk's model gave better volume prediction for softwood species, while the model by Max and Burkhart was superior for hardwood species.

Generally, all the stem profile equations over-estimated diameters and volumes to some extent due to the retransformation of squared diameter to diameter (Czaplewski and Bruce, 1990). This over-estimation becomes more apparent when the diameters of trees get larger. This is especially true for some species such as black spruce and balsam fir. However, for the range normally observed in Ontario for these species, this over-estimation was usually less than 10 percent.

RECOMMENDATIONS FOR FUTURE WORK

Due to the limitations of the data sets available, the single-entry volume table initiative was limited to the two northern regions of Ontario. To complete the above task for the whole province, further work is needed to collect data from the other two regions (Central and Southern), and continue the modelling process using the methodology demonstrated in this thesis. For some species with insufficient sample data and for other species without any data, additional or new data is required to adequately fit the height prediction models and the age prediction models.

For some tree species, the variation of height within a region might be too large. It would be desirable to produce sub-regional single-entry volume tables (such as forest site regions defined by Hills (1958)) when the variation of height-diameter relationships within a region proves significant, provided that the appropriate data sets for the subregions are available.

All the stem profile models were fitted by using data from the FMI data bank, new stem analysis data is also required to make these equations more adequate and more reliable. For species such as black spruce, balsam fir, white birch and white cedar, additional stem analysis data is needed. It is recommended that the stem profile models by regions and species be fitted when the data sets are made available.

In this project, the wood product mix model was basically for the estimation of various wood products for both a single tree and a stand. Further work to develop a system to model primary breakdown of trees or stands for optimal solutions based on the concept of global optimization, in which overall mill flow, product mixes and other global criteria will be considered is recommended.

Actual data for merchantable stands was unavailable. The lack of real data sets was a handicap to model dbh distributions found in merchantable stands. These dbh distributions, when available, can be used by combining the wood product mix model to describe the various combinations of raw products to be generated from merchantable stands.

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APPENDICES

APPENDIX A LIST OF THE SPECIES STUDIED

Common name	Latin name	Species code
	Softwoods	
White cedar	*Thuja occidentalis* L.	CE
Balsam fir	*Abies balsamea* (L.) Mill.	BF
Jack pine	*Pinus banksiana* Lamb.	PJ
Red pine	*Pinus resinosa* Ait.	PR
White pine	*Pinus strobus* L.	PW
Black spruce	*Picea mariana* (Mill.)B.S.P.	SB
White spruce	*Picea glauca* (Moench)Voss	SW
Tamarack	*Larix laricina* (Du Roi) K. Koch	LA
	Hardwoods	
Black ash	*Fraxinus nigra* Marsh.	AB
Largetooth aspen	*Populus grandidentata* Michx.	AL
Aspen (gen.)	*Populus*	AS
Trembling aspen	*Populus tremuloides* Michx.	AT
Beech	*Fagus grandifolia* Ehrh.	BE
White birch	*Betula papyrifera* Marsh.	BW
Yellow birch	*Betula alleghaniensis* Britton.	BY
Red maple	*Acer rubrum* L.	MR
Sugar maple	*Acer saccharum* Marsh.	MS
Balsam poplar	*Populus balsamifera* L.	PB

APPENDIX B SITE CLASS ASSIGNMENT EQUATIONS FOR MAJOR TREE SPECIES IN ONTARIO*

No.	Equations	Suitable Species
1	$scX=exp(4.050409-48.36061*AGE^{-1.0})$ $sc1=exp(3.8513-50.0826*AGE^{-1.0})$ $sc2=exp(3.501411-52.2586*AGE^{-1.0})$ $sc3=exp(4.2542-16.0182*AGE^{-0.5})$ $sc4=exp(4.2371-20.42529*AGE^{-0.5})$	Red pine, White pine
2	$scX=exp(3.823071-22.44245*AGE^{-1.0})$ $sc1=exp(3.6544-24.463*AGE^{-1.0})$ $sc2=exp(3.7298-14.223*AGE^{-0.75})$ $sc3=exp(3.6209-16.136*AGE^{-0.75})$ $sc4=exp(3.4644546-20.58747*AGE^{-0.75})$	Balsam poplar, Aspen (gen.)
3	$scX=exp(3.647108-27.9734*AGE^{-1.0})$ $sc1=exp(3.470401-24.463*AGE^{-1.0})$ $sc2=exp(3.392001-31.516*AGE^{-1.0})$ $sc3=exp(3.169538-32.53107*AGE^{-1.0})$ $sc4=exp(2.913733-38.07934*AGE^{-1.0})$	Black ash, Red oak Black cherry, Elm, White birch, Yellow birch, Red maple, Sugar maple
4	$scX=exp(3.747122-11.98245*AGE^{-0.75})$ $sc1=exp(3.4052-25.105*AGE^{-1.0})$ $sc2=exp(3.269-27.413*AGE^{-1.0})$ $sc3=exp(3.1146-30.748*AGE^{-1.0})$ $sc4=exp(2.881159-38.87831*AGE^{-1.0})$	Jack pine, Scots pine, Trembling aspen, White spruce
5	$scX=exp(3.575054-16.94687*AGE^{-0.75})$ $sc1=exp(3.4659-20.05*AGE^{-0.75})$ $sc2=exp(3.3579-23.78*AGE^{-0.75})$ $sc3=exp(3.4109-32.713*AGE^{-0.75})$ $sc4=exp(3.183022-37.27952*AGE^{-0.75})$	Balsam fir, White cedar, Black spruce

* From Maurer (1993)

APPENDIX C LIST OF STEM PROFILE MODELS

The four stem profile models tested in this study are listed bellow. According to Martin (1981), each model is presented by: (a) basic stem profile equation, (b) height prediction equation, and (c) volume prediction equation.

To simplify the text, the use of the following notations will be consistent in this appendix. Variables not listed are defined where they are used.

d: top diameter inside bark at height h,

D: diameter outside bark at breast height (1.3 m),

h: height above the ground to top diameter d,

H: total tree height from ground to tip,

H_L: lower limit of tree height for volume calculation,

H_U: upper limit of tree height for volume calculation,

V_L: log volume of tree excluding bark between any two height limits,

b_i, c_i: regression coefficients peculiar to specific equations.

1. Demaerschalk's (1972) model

$$\frac{d^2}{D^2} = (10^{2b_1})\,(D^{(2b_2-2)})\,[(H-h)^{2b_3}]\,(H^{2b_4}) \qquad [C1.a]$$

$$h = H - \left[\frac{d}{(10^{b_1})\ (D^{b_2})\ (H^{b_4})}\right]^{\frac{1}{b_3}} \qquad [C1.b]$$

$$V_L = \frac{(0.0000785)\ (10^{2b_1})\ (D^{2b_2})\ (H^{2b_4})\ (X_1^Z - X_2^Z)}{Z} \qquad [C1.c]$$

Where:

$Z = 2\ b_3 + 1$

$X_1 = H - H_L$

$X_2 = H - H_U$

2. Max and Burkhart's (1976) model

$$\frac{d^2}{D^2} = b_1\left(\frac{h}{H}-1\right) + b_2\left(\frac{h^2}{H^2}-1\right) + b_3\left(a_1-\frac{h}{H}\right)^2 I_1 + b_4\left(a_2-\frac{h}{H}\right)^2 I_2 \qquad [C2.a]$$

Where:

a_i = join points; the upper point is i=1,
the lower point is i=2

$I_1 = 1$, if $h/H \leq a_1$

$= 0$, otherwise

$I_2 = 1$, if $h/H \leq a_2$

$= 0$, otherwise

$$h = \frac{H}{2\,A_3}\left(-A_2 - \sqrt{A_2^2 - 4\,A_1\,A_3}\right) \qquad [C2.b]$$

Where:

$A_1 = -b_1 - b_2 - d^2/D^2 + I_1'a_1^2b_3 + I_2'a_2^2b_4$

$A_2 = b_1 - 2I_1'a_1b_3 - 2I_2'a_2b_4$

$A_3 = b_2 + I_1'b_3 + I_2'b_4$

$I_1' = 1$, if $d \geq d_1$,

$= 0$, otherwise.

$I_2' = 1$, if $d \geq d_2$,

$=0$, otherwise.

d_1 = estimated diameter at height a_1H

$$= D\sqrt{b_1(a_1-1) + b2(a_1^2-1)}$$

d_2 = estimated diameter at height a_2H

$$= D\sqrt{b1(a_2-1) + b_2(a_2^2-1) + b_3(a_1-a_2)^2}$$

$$V_L = 0.0000785\,D^2\,H\,(B_1 + B_2 - B_3 - B_4 - B_5) \qquad [C2.c]$$

Where:

$$B_1 = \frac{b_2}{3}\left[\left(\frac{H_U}{H}\right)^3 - \left(\frac{H_L}{H}\right)^3\right]$$

$$B_2 = \frac{b_1}{2}\left[\left(\frac{H_U}{H}\right)^2 - \left(\frac{H_L}{H}\right)^2\right]$$

$$B_3 = (b_1 + b_2)\left[\left(\frac{H_U}{H}\right) - \left(\frac{H_L}{H}\right)\right]$$

$$B_4 = \frac{b_3}{3}\left[\left(a_1 - \frac{H_U}{H}\right)^3 I_1 - \left(a_1 - \frac{H_L}{H}\right)^3 J_1\right]$$

$$B_5 = \frac{b_4}{3}\left[\left(a_2 - \frac{H_U}{H}\right)^3 I_2 - \left(a_2 - \frac{H_L}{H}\right)^3 J_2\right]$$

$I_1 = 1$, if $H_U/H \le a_1$

$= 0$, otherwise

$I_2 = 1$, if $H_U/H \le a_2$

$= 0$, otherwise

$J_1 = 1$, if $H_L/H \le a_1$

$= 0$, otherwise

$J_2 = 1$, if $H_L/H \le a_2$

$= 0$, otherwise

3. Kozak *et al.* (1969) model

$$\frac{d^2}{D^2} = b_1\left(\frac{h}{H} - 1\right) + b_2\left(\frac{h^2}{H^2} - 1\right) \qquad [C3.a]$$

$$h = \frac{(-b_1H) - \sqrt{(b_1H)^2 - 4b_2\ (b_0H^2 - \frac{d^2H^2}{D^2})}}{2\ b_2} \qquad [C3.b]$$

$$V_L = (0.0000785D^2)\left[b_0(H_U-H_L) + \frac{b_1(H_U{}^2-H_L{}^2)}{2\ H} + \frac{b_2(H_U{}^3-H_L{}^3)}{3\ H^2}\right] \qquad [C3.c]$$

Where:

$$b_0 = -b_1 - b_2$$

4. Ormerod's (1973) model

$$\frac{d^2}{D^2} = \left(\frac{H-h}{H-1.3}\right)^{2b_1} \qquad [C4.a]$$

$$h = H - \left[\left(\frac{d}{D}\right)^{\frac{1}{b_1}}(H - 1.3)\right] \qquad [C4.b]$$

$$V_L = \frac{(0.0000785D^2)(H-1.3)}{Y}\left[\left(\frac{H-H_L}{H-1.3}\right)^Y - \left(\frac{H-H_U}{H-1.3}\right)^Y\right] \qquad [C4.c]$$

Where:

$$Y = 2\ b_1 + 1$$

APPENDIX D FORTRAN PROGRAM TO CALCULATE THE WOOD PRODUCT MIXES FOR STAND

This program was written to facilitate the calculations for the wood product mixes based on dbh class distributions of a stand. The input data includes only species, dbh classes and number of stems per dbh class. The output includes: volumes for various wood products for each dbh class and species, and the total for each species and stand. The heights in the program are calculated by the corresponding regional or provincial pooled-site single-entry volume equations. If site classes are available, the site-stratified single-entry volume equations can be used to calculate the height for each dbh class. Top diameters and section heights in the program are calculated by Max and Burkhart's stem profile equations, while all the volume calculations are made by the stem profile equations by Demaerschalk.

The specifications of wood products were taken from Northern Wood Preservers Inc., Thunder Bay, Ontario. Four types of products were included, i.e., veneer logs, pulp logs, saw logs and chips. Because of the similar dimensional requirements for saw logs and pulp logs, the output of these two products can be assigned either way depending on the product preference. Basically, chips take everything.

Since the dimensional requirements of wood products vary from case to case, and from time to time, minor adjustment is needed according to specific product specifications.

PARAMETERS AND VARIABLES DESCRIPTION

n1	number of diameter classes for all the species in the stand
n2	number of species in the stand
minven	minimum diameter requirement for veneer logs
maxven	maximum diameter requirement for veneer logs
minsaw	minimum diameter requirement for saw logs
minpulp	minimum diameter requirement for pulp logs
maxpulp	maximum diameter requirement for pulp logs
c	constant in the calculation of log volume for the metric system
stem	number of stems for given dbh class

no	series number assigned to each species in the coefficient file (wood.coe)
sppstem	total stems for given species in the stand
stdstem	total stems for the stand
spp1	species' name in the stand data file (stand.dat)
spp2	species' name in the coefficient file (wood.coe)
dbh	dbh class
ht	calculated total height for dbh by Equation [1]
voltre	calculated tree volume for dbh by Equation [C1.c]
volcls	total volume for dbh class
ven1	average volume of veneer logs for dbh class
saw1	average volume of saw logs for dbh class
pulp1	average volume of pulp logs for dbh class
chip1	average volume of chips for dbh class
ven2	total volume of veneer logs for dbh class
saw2	total volume of saw logs for dbh class
pulp2	total volume of pulp logs for dbh class
chip2	total volume of chips for dbh class
dstp	diameter at stump height
h1	coefficient b1 in Equation [1]
h2	coefficient b2 in Equation [1]
a1	upper join point a1 in Equation [C2.a]
a2	upper join point a2 in Equation [C2.a]
b1	coefficient b1 in Equation [C2.a]
b2	coefficient b2 in Equation [C2.a]
b3	coefficient b3 in Equation [C2.a]
b4	coefficient b4 in Equation [C2.a]
d1	coefficient b1 in Equation [C1.a]
d2	coefficient b2 in Equation [C1.a]
d3	coefficient b3 in Equation [C1.a]
d4	coefficient b4 in Equation [C1.a]
slen	log lengths requirements of veneer logs, saw logs and pulp logs
hvmin	height limit at minven for the volume calculation of veneer logs
hsmin	height limit at minsaw for the volume calculation of saw logs
hpmin	height limit at minpulp for the volume calculation of pulp logs
hvmax	height limit at maxven for the volume calculation of veneer logs
hpmax	height limit at maxpulp for the volume calculation of pulp logs
yvmin1	number of slen for the volume calculation of veneer logs in the case of minimum diameter requirement
yvmin2	actual length qualified for the volume calculation of veneer logs in the case of minimum diameter requirement

yvmax1	number of slen for the volume calculation of veneer logs in the case of maximum diameter requirement
yvmax2	actual length qualified for the volume calculation of veneer logs in the case of maximum diameter requirement
ysmin1	number of slen for the volume calculation of saw logs in the case of maximum diameter requirement
ysmin2	actual length qualified for the volume calculation of saw logs in the case of maximum diameter requirement
ypmin1	number of slen for the volume calculation of pulp logs in the case of minimum diameter requirement
ypmin2	actual length qualified for the volume calculation of pulp logs in the case of minimum diameter requirement
ypmax1	number of slen for the volume calculation of pulp logs in the case of maximum diameter requirement
ypmax2	actual length qualified for the volume calculation of pulp logs in the case of maximum diameter requirement
stdven	total volume of veneer logs for the stand
stdsaw	total volume of saw logs for the stand
stdpulp	total volume of pulp logs for the stand
stdchip	total volume of chips for the stand
stdvol	total volume for the stand
wavedbh	weighted average dbh for the stand
waveht	weighted average height for the stand
wavetre	weighted average tree volume for the stand
sppven	total volume of veneer logs for given species in the stand
sppsaw	total volume of saw logs for given species in the stand
spppulp	total volume of pulp logs for given species in the stand
sppchip	total volume of chips for given species in the stand
sppvol	total volume for given species in the stand
waspdbh	weighted average dbh for given species in the stand
waspht	weighted average height for given species in the stand
wasptre	weighted average tree volume for given species in the stand

```
                    *** PARAMETER ENTRANCE ***

        parameter(n1=74,n2=13,minven=23,maxven=51,minsaw=11,
       $          minpulp=10,maxpulp=41,slen=2.54,c=.0000785)


       *** VARIABLE DEFINITION AND STORAGE ALLOCATION ***


        character*2 spp1(n1),spp2(n2)
        integer stem(n1),sppstem(n2),stdstem,no(n2)
        real dbh(n1),ht(n1),voltre(n1),volcls(n1),ven1(n1),
       $    ven2(n1),saw1(n1),saw2(n1),pulp1(n1),pulp2(n1),
       $    chip1(n1),chip2(n1),dstp(n1),hvmin(n1),hsmin(n1),
       $    hpmin(n1),hvmax(n1),hpmax(n1),yvmin1(n1),
       $    yvmin2(n1),yvmax1(n1),yvmax2(n1),ypmin1(n1),
       $    ypmin2(n1),ypmax1(n1),ypmax2(n1),ysmin1(n1),
       $    ysmin2(n1),h1(n2),h2(n2),a1(n2),a2(n2),
       $    b1(n2),b2(n2),b3(n2),b4(n2),d1(n2),d2(n2),
       $    d3(n2),d4(n2),sppdbh(n2), sppht(n2),spptre(n2),
       $    waspdbh(n2),waspht(n2),wasptre(n2),sppvol(n2),
       $    sppven(n2),sppsaw(n2),spppulp(n2),sppchip(n2)


                     *** OPEN I/O UNITS ***


        open(1,file='stand.dat',status='old')
        open(2,file='wood.coe',status='old')
        open(3,file='wood.out',status='new')


      *** READ THE STAND FILE AND THE COEFFICIENT FILE ***


        do i=1,n1
           read(1,*)spp1(i),dbh(i),stem(i)
        end do
        do i=1,n2
           read(2,*)no(i),spp2(i),h1(i),h2(i),a1(i),a2(i),
       $             b1(i),b2(i),b3(i),b4(i),d1(i),d2(i),
       $             d3(i),d4(i)
        end do
```

```
*** CALCULATE THE VOLUMES FOR VARIOUS PRODUCTS ***
  *** FOR EACH SPECIES AND DBH CLASS BASED ON ***
      *** THE DIMENSIONAL REQUIREMENTS ***


  do i=1,n1
     do k=1,n2
        if(spp1(i) .eq. spp2(k))j=no(k)
     end do
     ht(i)=1.3+h1(j)*exp(h2(j)/dbh(i))
     voltre(i)=(c*10**(2*d1(j))*dbh(i)**(2*d2(j))*
$              ht(i)**(2*d4(j))*ht(i)**(2*d3(j)+1))
$              /(2*d3(j)+1)
     volcls(i)=voltre(i)*stem(i)
     dstp(i)=dbh(i)*sqrt(b1(j)*(0-1)+b2(j)*(0-1)+
$            b3(j)*a1(j)**2+b4(j)*a2(j)**2)
     dd1=dbh(i)*sqrt(b1(j)*(a1(j)-1)+
$        b2(j)*(a1(j)*a1(j)-1))
     dd2=dbh(i)*sqrt(b1(j)*(a2(j)-1)+
$        b2(j)*(a2(j)*a2(j)-1)+b3(j)*(a1(j)-a2(j))**2)
     if(spp1(i) .eq. 'sb' .or. spp1(i) .eq. 'sw')then
        if(minven .ge. dd1)ivmin1=1
        if(minven .lt. dd1)ivmin1=0
        if(minven .ge. dd2)ivmin2=1
        if(minven .lt. dd2)ivmin2=0
        xvmin1=-b1(j)-b2(j)-((minven/dbh(i))**2)+
$              ivmin1*a1(j)*a1(j)*b3(j)+
$              ivmin2*a2(j)*a2(j)*b4(j)
        xvmin2=b1(j)-2*ivmin1*a1(j)*b3(j)-
$              2*ivmin2*a2(j)*b4(j)
        xvmin3=b2(j)+ivmin1*b3(j)+ivmin2*b4(j)
        root=xvmin2**2-4*xvmin1*xvmin3
        hvmin(i)=ht(i)*(-xvmin2-sqrt(root))/(2*xvmin3)
        if(dstp(i) .lt. minven)hvmin(i)=0
        yvmin1(i)=int(hvmin(i)/slen)
        yvmin2(i)=yvmin1(i)*slen
        ven1(i)=(c*10**(2*d1(j))*dbh(i)**(2*d2(j))*
$               ht(i)**(2*d4(j))*((ht(i)-0)**
$               (2*d3(j)+1)-(ht(i)-yvmin2(i))**
$               (2*d3(j)+1)))/(2*d3(j)+1)
        if(maxven .ge. dd1)ivmax1=1
        if(maxven .lt. dd1)ivmax1=0
        if(maxven .ge. dd2)ivmax2=1
        if(maxven .lt. dd2)ivmax2=0
        xvmax1=-b1(j)-b2(j)-((maxven/dbh(i))**2)+
$              ivmax1*a1(j)*a1(j)*b3(j)+
$              ivmax2*a2(j)*a2(j)*b4(j)
        xvmax2=b1(j)-2*ivmax1*a1(j)*b3(j)-
$              2*ivmax2*a2(j)*b4(j)
```

```
         xvmax3=b2(j)+ivmax1*b3(j)+ivmax2*b4(j)
         root=xvmax2**2-4*xvmax1*xvmax3
         hvmax(i)=ht(i)*(-xvmax2-sqrt(root))/(2*xvmax3)
         if(dstp(i) .lt. maxven)hvmax(i)=0
         yvmax1(i)=int(hvmax(i)/slen)
         yvmax2(i)=yvmax1(i)*slen
         saw1(i)=(c*10**(2*d1(j))*dbh(i)**(2*d2(j))
$                *ht(i)**(2*d4(j))*((ht(i)-0)**
$                (2*d3(j)+1)-(ht(i)-yvmax2(i))**
$                (2*d3(j)+1)))/(2*d3(j)+1)
         saw2(i)=saw1(i)*stem(i)
         ven1(i)=ven1(i)-saw1(i)
         ven2(i)=ven1(i)*stem(i)
         if(minpulp .ge. dd1)ip1=1
         if(minpulp .lt. dd1)ip1=0
         if(minpulp .ge. dd2)ip2=1
         if(minpulp .lt. dd2)ip2=0
         xpmin1=-b1(j)-b2(j)-((minpulp/dbh(i))
$                **2)+ip1*a1(j)*a1(j)*b3(j)+
$                ip2*a2(j)*a2(j)*b4(j)
         xpmin2=b1(j)-2*ip1*a1(j)*b3(j)-
$                2*ip2*a2(j)*b4(j)
         xpmin3=b2(j)+ip1*b3(j)+ip2*b4(j)
         root=xpmin2**2-4*xpmin1*xpmin3
         hpmin(i)=ht(i)*(-xpmin2-sqrt(root))/(2*xpmin3)
         if(dstp(i) .lt. minpulp)hpmin(i)=0
         ypmin1(i)=int(hpmin(i)/slen)
         ypmin2(i)=ypmin1(i)*slen
         pulp1(i)=(c*10**(2*d1(j))*dbh(i)**
$                  (2*d2(j))*ht(i)**(2*d4(j))*
$                  ((ht(i)-0)**(2*d3(j)+1)-(ht(i)-
$                 ypmin2(i))**(2*d3(j)+1)))/(2*d3(j)+1)
         pulp1(i)=pulp1(i)-ven1(i)-saw1(i)
         pulp2(i)=pulp1(i)*stem(i)
         chip1(i)=voltre(i)-ven1(i)-saw1(i)-pulp1(i)
         chip2(i)=chip1(i)*stem(i)
      else if(spp1(i) .eq. 'pj' .or. spp1(i) .eq. 'pr'
$           .or. spp1(i) .eq. 'pw' .or. spp1(i) .eq.
$           'bf')then
         if(spp1(i) .ne. 'bf')then
           if(minpulp .ge. dd1)ip1=1
           if(minpulp .lt. dd1)ip1=0
           if(minpulp .ge. dd2)ip2=1
           if(minpulp .lt. dd2)ip2=0
           xpmin1=-b1(j)-b2(j)-((minpulp/dbh(i))**2)+
$                 ip1*a1(j)*a1(j)*b3(j)+
$                 ip2*a2(j)*a2(j)*b4(j)
           xpmin2=b1(j)-2*ip1*a1(j)*b3(j)-
$                 2*ip2*a2(j)*b4(j)
```

```
              xpmin3=b2(j)+ip1*b3(j)+ip2*b4(j)
              root=xpmin2**2-4*xpmin1*xpmin3
              hpmin(i)=ht(i)*(-xpmin2-sqrt(root))
     $                 /(2*xpmin3)
              if(dstp(i) .lt. minpulp)hpmin(i)=0
              ypmin1(i)=int(hpmin(i)/slen)
              ypmin2(i)=ypmin1(i)*slen
              pulp1(i)=(c*10**(2*d1(j))*dbh(i)**(2*d2(j))*
     $                 ht(i)**(2*d4(j))*((ht(i)-0)**
     $                 (2*d3(j)+1)-(ht(i)-ypmin2(i))**
     $                 (2*d3(j)+1)))/(2*d3(j)+1)
              if(maxpulp .ge. dd1)ipmax1=1
              if(maxpulp .lt. dd1)ipmax1=0
              if(maxpulp .ge. dd2)ipmax2=1
              if(maxpulp .lt. dd2)ipmax2=0
              xpmax1=-b1(j)-b2(j)-((maxpulp/dbh(i))**2)+
     $               ipmax1*a1(j)*a1(j)*b3(j)+ipmax2*a2(j)*
     $               a2(j)*b4(j)
              xpmax2=b1(j)-2*ipmax1*a1(j)*b3(j)-
     $               2*ipmax2*a2(j)*b4(j)
              xpmax3=b2(j)+ipmax1*b3(j)+ipmax2*b4(j)
              root=xpmax2**2-4*xpmax1*xpmax3
              hpmax(i)=ht(i)*(-xpmax2-sqrt(root))
     $                 /(2*xpmax3)
              if(dstp(i) .lt. maxpulp)hpmax(i)=0
              ypmax1(i)=int(hpmax(i)/slen)
              ypmax2(i)=ypmax1(i)*slen
              saw1(i)=(c*10**(2*d1(j))*dbh(i)**
     $                (2*d2(j))*ht(i)**(2*d4(j))*((ht(i)-0)
     $                **(2*d3(j)+1)-(ht(i)-ypmax2(i))
     $                **(2*d3(j)+1)))/(2*d3(j)+1)
              saw2(i)=saw1(i)*stem(i)
              pulp1(i)=pulp1(i)-saw1(i)
              pulp2(i)=pulp1(i)*stem(i)
              chip1(i)=voltre(i)-pulp1(i)-saw1(i)
              chip2(i)=chip1(i)*stem(i)
            else
              if(minsaw .ge. dd1)is1=1
              if(minsaw .lt. dd1)is1=0
              if(minsaw .ge. dd2)is2=1
              if(minsaw .lt. dd2)is2=0
              xsmin1=-b1(j)-b2(j)-((minsaw/dbh(i))**2)+
     $               is1*a1(j)*a1(j)*b3(j)+
     $               is2*a2(j)*a2(j)*b4(j)
              xsmin2=b1(j)-2*is1*a1(j)*b3(j)-
     $               2*is2*a2(j)*b4(j)
              xsmin3=b2(j)+is1*b3(j)+is2*b4(j)
              root=xsmin2**2-4*xsmin1*xsmin3
              hsmin(i)=ht(i)*(-xsmin2-sqrt(root))
```

```
     $                 /(2*xsmin3)
              if(dstp(i) .lt. minsaw)hsmin(i)=0
              ysmin1(i)=int(hsmin(i)/slen)
              ysmin2(i)=ysmin1(i)*slen
              saw1(i)=(c*10**(2*d1(j))*dbh(i)
     $                **(2*d2(j))*ht(i)**(2*d4(j))*
     $                ((ht(i)-0)**(2*d3(j)+1)-(ht(i)-
     $                ysmin2(i))**(2*d3(j)+1)))/(2*d3(j)+1)
              saw2(i)=saw1(i)*stem(i)
              chip1(i)=voltre(i)-saw1(i)
              chip2(i)=chip1(i)*stem(i)
            end if
         else
            chip1(i)=voltre(i)
            chip2(i)=chip1(i)*stem(i)
         end if
         do k=1,n2
            sppdbh(j)=sppdbh(j)+dbh(i)*stem(i)
            sppht(j)=sppht(j)+ht(i)*stem(i)
            spptre(j)=spptre(j)+voltre(i)*stem(i)
            sppstem(j)=sppstem(j)+stem(i)
            sppvol(j)=sppvol(j)+volcls(i)
            sppven(j)=sppven(j)+ven2(i)
            sppsaw(j)=sppsaw(j)+saw2(i)
            spppulp(j)=spppulp(j)+pulp2(i)
            sppchip(j)=sppchip(j)+chip2(i)
         end do
      end do


*** CALCULATE THE SUMMARY INFORMATION FOR EACH SPECIES ***


      do k=1,n2
         sppdbh(k)=sppdbh(k)/n2
         sppht(k)=sppht(k)/n2
         spptre(k)=spptre(k)/n2
         sppstem(k)=sppstem(k)/n2
         sppvol(k)=sppvol(k)/n2
         sppven(k)=sppven(k)/n2
         sppsaw(k)=sppsaw(k)/n2
         spppulp(k)=spppulp(k)/n2
         sppchip(k)=sppchip(k)/n2
      end do
      do k=1,n2
         if(sppstem(k) .eq. 0)goto 10
            waspdbh(k)=sppdbh(k)/sppstem(k)
            waspht(k)=sppht(k)/sppstem(k)
```

```
            wasptre(k)=spptre(k)/sppstem(k)
10         continue
      end do


  *** CALCULATE THE SUMMARY INFORMATION FOR THE STAND ***


      do i=1,n1
         stddbh=stddbh+dbh(i)*stem(i)
         stdht=stdht+ht(i)*stem(i)
         stdtre=stdtre+voltre(i)*stem(i)
         stdstem=stdstem+stem(i)
         stdvol=stdvol+volcls(i)
         stdven=stdven+ven2(i)
         stdsaw=stdsaw+saw2(i)
         stdpulp=stdpulp+pulp2(i)
         stdchip=stdchip+chip2(i)
      end do
      wavedbh=stddbh/stdstem
      waveht=stdht/stdstem
      wavetre=stdtre/stdstem



           *** WRITE THE RESULTS INTO A FILE ***


      do i=1,n1
         write(3,100)spp1(i),dbh(i),ht(i),voltre(i),
     $               stem(i),volcls(i),ven2(i),saw2(i),
     $               pulp2(i),chip2(i)
      end do
      write(3,*)
      write(3,200)
      write(3,*)
      do k=1,n2
         if(sppstem(k) .eq. 0)goto 20
            write(3,100)spp2(k),waspdbh(k),waspht(k),
     $                  wasptre(k),sppstem(k),sppvol(k),
     $                  sppven(k),sppsaw(k),spppulp(k),
     $                  sppchip(k)
20       continue
      end do
      write(3,*)
      write(3,300)
      write(3,*)
      write(3,400)wavedbh,waveht,wavetre,stdstem,stdvol,
```

```
     $              stdven,stdsaw,stdpulp,stdchip
100   format(1x,1x,a2,1x,f6.2,1x,f5.2,1x,f7.4,1x,i5,1x,
     $        5(f8.4,1x))
200   format(1x,'Sum By Species')
300   format(1x,'Sum For Stand')
400   format(1x,'ALL',1x,f6.2,1x,f5.2,1x,f7.4,1x,i5,1x,
     $        5(f8.4,1x))
```

*** TERMINATION ***

```
      close(1)
      close(2)
      close(3)
      end
```

APPENDIX E PARAMETERS ESTIMATED FOR HEIGHT PREDICTION EQUATIONS FOR NORTHWESTERN ONTARIO

Species	Site Class	n	b1	b2	MSE	R^2
BF	X	707	22.818273263	-9.305068218	3.6691	.6959
	1	249	18.716505098	-8.292207951	3.1295	.6466
	2	80	17.321345520	-9.208221703	2.3866	.7661
BW	X	54	26.297706997	-7.546553657	8.7855	.7258
	1	120	22.702191598	-6.638045700	3.3614	.7178
	2	278	21.958976921	-7.198197907	3.2973	.7596
	3	205	19.431364659	-6.802603434	2.9670	.7626
	4	56	14.988555192	-6.369777598	3.5229	.3945
CE	all	108	20.852090879	-13.28011589	3.8347	.6328
PB	all	97	30.950104918	-13.14854045	6.9557	.5594
PJ	X	126	34.632608581	-13.52132614	7.6948	.7171
	1	701	29.265315093	-10.74093509	4.2500	.6774
	2	728	24.513127228	-8.562328324	2.8267	.5984
	3	289	21.514253897	-8.447308384	2.7316	.5511
	4	211	17.987117727	-10.14030491	2.7699	.8525
PO	X	48	37.050960011	-11.42473368	6.5611	.8651
	1	304	34.086275614	-10.53863669	6.0925	.7576
	2	572	29.098611992	-9.037323405	4.1126	.7831
	3	376	24.131662518	-7.934089579	4.1650	.7434
	4	82	19.125505467	-7.345334345	4.7123	.5987
PR	all	255	31.122271305	-14.89367610	8.9824	.3273
PW	all	156	36.636781025	-21.67734865	12.1348	.5569
SB	X	1207	31.982456506	-13.76392665	5.5905	.7225
	1	843	25.378487459	-10.80154231	4.4024	.6238
	2	368	23.349415943	-10.20145156	3.5258	.6282
	3	200	20.603962286	-8.748798714	2.3506	.5227
	4	67	18.206273560	-8.438616673	1.9438	.6151
SW	X	66	35.078552868	-13.71269628	8.2368	.6154
	1	177	28.241795450	-11.78307572	5.0150	.7387
	2	160	24.867107680	-11.78806361	3.7519	.7561
	3	142	18.946863913	-8.501655304	3.4997	.6676
	4	44	19.080465571	-11.17130907	3.0038	.5872

APPENDIX F PARAMETERS ESTIMATED FOR HEIGHT PREDICTION EQUATIONS FOR NORTHEASTERN ONTARIO

Species	Site Class	n	b1	b2	MSE	R^2
AB	all	19	27.105310473	-12.87520504	3.2974	.6847
AT	X	73	39.848042264	-12.04180321	9.2977	.7399
	1	436	33.292827450	-9.971991437	5.5881	.5993
	2	574	29.026879221	-9.415972261	4.4864	.5838
	3	285	26.020787522	-9.397394270	4.7240	.6232
	4	36	18.039105995	-6.174607315	2.3088	.6081
BF	X	338	24.616877453	-10.32802211	4.0635	.6254
	1	168	19.237259645	-9.122497460	2.1395	.7565
	2	82	17.659089963	-9.395660795	1.6601	.8095
	3	18	14.542580553	-8.969533426	3.1772	.6233
BW	X	75	31.007696108	-9.946290239	4.8776	.7175
	1	220	26.788705122	-9.022286644	3.3678	.5974
	2	422	22.956840332	-8.262002419	2.8551	.5895
	3	262	20.391925794	-8.318052225	3.1253	.6288
	4	44	16.482864837	-8.906881208	1.6473	.6834
BY	all	35	23.162540929	-11.66922779	4.5393	.4472
CE	X	30	24.238095916	-13.36555640	5.4862	.6965
	1	112	22.664895451	-15.62751807	3.5584	.6447
	2	131	19.454985994	-13.25067820	2.4338	.7226
	3	97	18.042273723	-13.85980607	2.3993	.7483
	4	22	18.120643850	-17.46896412	1.0230	.8076
LA	all	108	27.325001194	-11.27409620	5.5704	.7741
MS	all	18	26.566574936	-14.03096091	2.5842	.7850
MR	all	76	20.168867613	-7.312805668	4.4292	.5655
PB	all	80	27.466417990	-10.51183997	4.8366	.7176
PJ	X	643	31.334591113	-10.87619213	5.7133	.6461
	1	1048	24.682442456	-7.574868031	5.4309	.5503
	2	267	19.848218234	-5.305364509	5.6855	.2529
	3	68	18.398041518	-7.559071417	4.8732	.3166
	4	12	12.214712941	-4.893522516	2.9610	.2764
PR	all	20	29.193414089	-15.01969211	1.5388	.4650
PW	all	144	44.718891012	-25.49728345	9.4902	.7969
SB	X	1013	28.911560847	-12.41259173	5.5356	.6186
	1	643	26.164808352	-11.56642286	3.9313	.7257
	2	378	22.785412552	-10.22217594	3.1512	.7284
	3	109	19.653549109	-8.731535971	1.7093	.8066
	4	27	15.193740215	-7.154421233	1.1602	.7363
SW	X	58	39.055819399	-18.25027976	7.7796	.5077
	1	178	28.148559099	-11.82323001	2.9828	.5720
	2	143	24.936253768	-11.03818561	3.0320	.6668
	3	89	22.412100762	-11.71953303	1.8943	.7970
	4	16	14.630627874	-7.691622550	1.2442	.7351

APPENDIX G PARAMETERS ESTIMATED FOR STEM PROFILE MODELS

Appendix G-1 Parameters estimated for Demaerschalk's (1972) model

Species	Size	b1	b2	b3	b4	MSE	R^2
Largetooth aspen	592	-.049975	.970239	0.690464	-.617293	.00601	.94483
Aspen (gen.)	3175	-.043488	1.003737	0.578755	-.555715	.00350	.95686
Trembling aspen	854	.033251	1.035324	0.662901	-.718106	.00461	.96321
Beech	114	.029611	1.047708	0.724240	-.769283	.00513	.96582
Balsam fir	494	.069513	1.016638	0.610619	-.678979	.00472	.95904
White birch	591	.038528	1.020312	0.824839	-.847540	.00988	.94219
White cedar	802	.077032	1.063347	0.949528	-1.057839	.01031	.93136
Jack pine	3185	.065070	.970354	0.585574	-.602582	.00600	.94698
Red pine	4869	.103114	.958414	0.650413	-.676005	.00733	.94216
White pine	5254	.085613	.985592	0.693985	-.732518	.00785	.93987
Black spruce	1031	.095149	1.015739	0.708131	-.780004	.00839	.93988
White spruce	424	.145635	1.000732	0.919070	-1.002816	.01275	.92554

Appendix G-2 Parameters estimated for Max and Burkhart's (1976) model

Species	Size	a1	a2	b1	b2	b3	b4	MSE	R^2
Large. aspen	592	.73076	.13441	-4.21696	2.11106	-2.19091	9.91985	.0051	.95294
Aspen (gen.)	3175	.79147	.06141	-5.91138	2.96871	-3.26271	65.95177	.0028	.96616
Trembl. aspen	854	.72509	.08659	-4.04174	1.97654	-2.11324	19.20136	.0041	.96710
Beech	114	.80843	.10064	-6.48795	3.31302	-3.35558	31.68926	.0035	.97707
Balsam fir	494	.75000	.06000	-4.58296	2.21483	-2.47938	45.61874	.0046	.96019
White birch	591	.69187	.10679	-3.97912	2.00355	-2.07625	32.14399	.0067	.96106
White cedar	802	.78424	.07237	-3.80450	1.89441	-1.47351	144.22722	.0056	.96276
Jack pine	3185	.84883	.09964	-4.94986	2.28480	-2.42782	29.02209	.0042	.96315
Red pine	4869	.77652	.09197	-5.21237	2.55118	-2.89904	51.80430	.0054	.95708
White pine	5254	.67025	.04547	-3.77996	1.82266	-2.28883	301.61778	.0060	.95406
Black spruce	1031	.71397	.08635	-3.51982	1.61500	-1.88183	65.97321	.0045	.96765
White spruce	424	.68155	.06664	-3.25958	1.58589	-1.36665	129.72728	.0086	.95013

Appendix G-3 Parameters estimated for Kozak *et al* (1969) model

Species	Size	b1	b2	MSE	R^2
Largetooth aspen	592	-1.535420	.503360	.00607	.94406
Aspen (gen.)	3175	-1.196093	.217476	.00356	.95619
Trembl. aspen	854	-1.488934	.446002	.00479	.96166
Beech	114	-1.800841	.652085	.00502	.96594
Balsam fir	494	-1.374381	.313519	.00523	.95447
White birch	591	-2.054222	.871360	.00970	.94310
White cedar	802	-2.298352	1.086425	.01013	.93234
Jack pine	3185	-1.307885	.276449	.00610	.94601
Red pine	4869	-1.542600	.445216	.00845	.93332
White pine	5254	-1.642996	.534409	.00893	.93158
Black spruce	1031	-1.773200	.612695	.00854	.93866
White spruce	424	-2.347415	1.083625	.01304	.92352

Appendix G-4 Parameters estimated for Ormerod's (1973) model

Species	Size	b1	MSE	R^2
Largetooth aspen	592	.760652	.00816	.92472
Aspen (gen.)	3175	.662443	.00554	.93171
Trembling aspen	854	.716617	.00645	.94835
Beech	114	.716830	.00558	.96177
Balsam fir	494	.675254	.00712	.93785
White birch	591	.816980	.01018	.94014
White cedar	802	.951282	.01035	.93079
Jack pine	3185	.638243	.00711	.93710
Red pine	4869	.682922	.00823	.93507
White pine	5254	.722503	.00877	.93280
Black spruce	1031	.708058	.00900	.93532
White spruce	424	.876742	.01341	.92113